

# The Two Faces of Information\*

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## Abstract

Information is a double-edged sword. On the one hand, it allows to adjust individual resources in response to foreseen changes. On the other hand, it increases the volatility of markets and unforeseen changes. The positive face of information is strengthening *self-insurance* by reallocating resources while its negative face is weakening the use of *market insurance*. We capture these two faces of information in a standard macro-finance setting and show that agents acquire information without internalizing its negative face, which leads to an excessive use of information from a social standpoint. This inefficiency holds for plausible parameter values and is manifested in excessive market volatility.

## 1 Introduction

Information improves the allocation of resources and the efficiency of investment decisions. Knowing the demand of a product allows firms to better target the use of inputs and the scale of production. Knowing about the productivity of an input allows to choose the best combination of other inputs. These choices usually respond to a combination of both increasing the level and to reduce the volatility of

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production. This positive role of information is well-understood and constitutes the backbone of a large literature discussing the incentives to acquire information and its aggregation.

Even though less acknowledged, information is also critical on determining the volatility of markets and their provision of liquidity and insurance. A more recent literature, since the seminal work of Hirshleifer (1971), has highlighted this negative role of information in weakening insurance. Dang et al. (2014), for example, applies this insight to banking and shows that information induces fluctuations in the transaction value of assets and then it may be detrimental to the provision of liquidity and insurance in financial markets, justifying the existence of banks as secret keepers.

This paper puts on equal footing these dual roles of information in a standard macro-finance setting with investment and trade. We study the problem of atomistic agents that have an endowment of capital and have to decide their production scale by committing to labor choices. To introduce a market to transact capital we assume that a fraction of an individual agent's capital proves completely useless for the agent after committing to the scale of production, but it can sell it in financial markets to other agents in exchange for other agents' capital.

In our setting the productivity of capital is stochastic and agents are risk averse, so insurance is needed. There are two sources of insurance. One is *self-insurance*. An individual can use labor choices both to increase production and to insure against these fluctuations. Specifically, if agents are very risk averse, they would like to work a lot when their capital is not very productive in order to smooth their consumption. The other is *market-insurance*. An agent can sell her own capital to other agents and buy a basket of other agents' capital, achieving diversification and eliminating the fundamental productivity volatility from idiosyncratic risk. How much of other agent's capital the agent is able to buy, however, critically depends on the price at which the agent is able to sell its own capital, which depends on the market's information about the agent's capital.

We compare the individual and social welfare with and without an *information technology*. Under this technology the agent receives a signal about the productivity of all capital in the economy, this is the productivity of the own capital as well as the

productivity of capital of all other agents. Focusing on information as a technology (or language) to process and interpret all signals in the economy is a departure from the traditional use of information as signals, and it is critical to understand our self-market insurance trade-off.

In an environment in which all agents have access to this technology, they fully face the volatility of their own capital, either because they have to use it in production or because they have to sell it to other informed agents. Agents can, however, use their labor choices to mitigate the volatility coming from the capital stochastic nature. In an environment without access to the information technology the agents can sell their own capital at the average productivity for a diversified basket of others' capital, eliminating the fundamental, idiosyncratic, source of volatility. However, for the fraction of own capital not traded in the market, agents cannot adjust labor optimally. While labor choices provide insurance against foreseen circumstances, transacting capital in the market introduces unforeseen circumstances.

In other words, the benefit of information is being able to adjust labor as a response to *all capital* variation. The benefits of no information is eliminate the fundamental variation of *traded capital*. As risk aversion increases labor adjusts more to smooth consumption and then the benefit of information increases. As trading needs increase the benefits of no information also increases as there is more traded capital. This implies that information is less likely to be optimal when trading needs are large relative to risk aversion.

When information is a “language to decode signals” instead of signals that are free to decode, agents that choose to learn about their own capital cannot help but also learn about others' capital. Acquiring the information technology then affects the price of others' capital and creates an externality. In other words, while access to the information technology puts a single investor in a better position to choose her labor and scale of production it also contributes to increase the volatility of prices in capital markets. The implication is that in the aggregate investors are more efficient in making investment decisions but also more uncertain about their production when many other investors have acquired the information technology.

This information externality can be severe enough such that a benevolent planner

would rather dispense from agents having access to the information technology, even if the information technology is free and perfect (provides infinitely precise signals). We show that indeed the conditions for excessive information processing are satisfied under empirically plausible parameter configurations.

When information acquisition is costly, the aforementioned externality produces a complementarity in information acquisition; it is optimal to acquire information if and only if others acquire information, which leads to multiple equilibria. When an agent expects to trade a large fraction of its own capital, it becomes less concerned about the productivity of the own capital that will be used in production and more concerned about the price at which that capital will be sold. On the one hand, when no other agent acquires information, the price of the own capital simply reflects its expected fundamental value, without volatility, discouraging the agent to spend resources to acquire the information technology. On the other hand, when other agents acquire information, the capital price will reflect its fundamental value, introducing price volatility and increasing the individual incentives to acquire the information technology to better forecast the selling price of her own capital.

This multiplicity has welfare ramifications. Under certain conditions it is socially optimal for agents to produce information and self-insure. It may exist, however, an equilibrium with no information, self-insurance and inefficient investment. Similarly, there are conditions under which it is optimal for agents not to have information and exploit market insurance, but there may exist an equilibrium with information acquisition and an excessively volatile market. These possible results are relevant in understanding fluctuations and inefficiencies in financial markets. The first possibility captures inefficient booms of liquid assets with poor asset origination quality (as it seems to have happened previous to the recent U.S. financial crisis), while the second captures the inefficiency created by a dry-up of liquid assets when the system turns into excessive information acquisition (as it seems to have happened at the wake of the crisis).

Finally, we generalize our results to a situation in which the precision of information is finite, and show that there is a non-monotonicity of welfare on this precision. This result is relevant as it implies that under circumstances in which no informa-

tion is socially preferred to perfect and costless information, it still may be socially inferior to imperfect and costless information. Intuitively, an initial increase in the precision of information improves more the possibility to forecast the capital productivity than it does to increase the volatility of those productivities. These effects reverse once the precision of information gets large enough.

Our work relates to the large branch of banking literature that explores the role of information in rationalizing the existence and organization of financial intermediaries. Such literature either focuses on the beneficial role of information for reallocating resources and improving the quality and profitability of assets (such as the seminal papers of Leland and Pyle (1977), Campbel and Kracaw (1980), Bester (1985) and Diamond (1984 and 1991)), or in the detrimental role of information for the value of liabilities (more recently discussed by Gorton and Pennacchi (1990) and Dang et al. (2014)). This paper analyzes these two effects in a unified macro setting and studies the potential multiplicity, non-monotonicity and inefficiencies arising in such a unified setting.

While Goldstein and Yang (2015) and Dow, Goldstein, and Guembel (2017) study the feedback effects between investment and trading prices that generate strategic complementarities, we study the complementarity in information acquisition generated by the needs to trade, and its interaction with the welfare gains from investment reallocation.

Our work also relates to the more general discussion about complementarity on information acquisition and aggregation. In particular, Morris and Shin (2002) and Angeletos and Pavan (2007) study the social value of public information and its efficient use in the presence of externalities that are hardwired into payoffs. While their focus is on the role of public signals, ours is on the possibility of multiplicity when there is an information acquisition choice. While Colombo, Femminis, and Pavan (2014) also study information acquisition in the presence of strategic complementarities, we highlight the welfare implications of multiple equilibria. Gaballo (2016) shows that private information about the future can inefficiently increase volatility of market outcomes when agents learn from a public asset price. We also emphasize

the detrimental effect of information in increasing volatility, but our mechanism does not rely on the endogeneity of signals and it also arises with perfect information.

Finally, Barlevy and Veronesi (2000) study the multiplicity of equilibrium that arise because of information acquisition in financial markets, but their complementarity arises from the endogenous precision of information and not from the impact of precision on prices. Allen, Morris, and Shin (2006) and Hellwig and Veldkamp (2009) also study a setting in which the price of an asset depends on information aggregation in the economy. They do not consider, however, the role of trading on information acquisition. We differ from all these papers on the modeling of information as a technology, under which information, even when free and perfect, may be socially undesirable.

The welfare implications of information in financial markets have been explored by Kurlat and Veldkamp (2015) as well. While they assume CARA preferences (standard in the finance literature) and explore a trade-off between the risk and return of different asset combinations, we assume CRRA preferences (more standard in the macroeconomics literature) and explore a trade-off between investment efficiency and liquidity provision. As with the previous papers, their paper is also based on the modeling of information as signals.

Finally, our work relates to the more recent literature that highlights the relevance of studying origination and trading of assets in a single setting. Vanasco (2017) shows that information acquisition at origination deepens asymmetric information and may lead to a freeze in trading of assets, with a collapse of liquidity. In our setting the issue does not come from asymmetric information in decentralized secondary markets, but rather by the aggregation of information in centralized secondary markets. Caramp (2017) also studies the role of liquidity on the incentives to originate poor quality assets, highlighting the reverse direction of liquidity affecting the quality of assets. While his focus is not on information, his paper is also an example of the close links between investments incentives and trading.

The next section presents the benchmark model in which information is free and perfect (infinite precision of signals) and in Section 3 we discuss the conditions and

sources of inefficiency of such information. Section 4 extends the analysis to imperfect information (finite precision of signals) and discusses the non-monotonicity of information precision on welfare. Section 5 extends the analysis to costly information and discusses the potential multiplicity of equilibria and its welfare implications. Section 6 concludes. In the Appendix we provide most proofs.

## 2 Model

There is a single period with a continuum of agents of mass one indexed by  $i \in (0, 1)$ . Agent  $i \in (0, 1)$  has utility function

$$U_i \equiv E_i \left[ \frac{C_i^{1-\sigma}}{1-\sigma} - L_i \right], \quad (1)$$

where  $C_i$  and  $L_i$  are consumption and labor specific to agent  $i$ ,  $\sigma$  is a constant relative risk-aversion parameter and  $E_i[\cdot]$  is an expectation operator conditional on the agent's information set  $\Omega_i$ . Each agent is endowed with one unit of *raw capital* denoted by  $x_i = 1$  to which it is associated an idiosyncratic stochastic productivity  $\theta_i \sim N(0, 1)$ , independently distributed across agents.

The timing of the period can be divided into four stages. At stage one, each agent chooses whether or not to become *perfectly informed* about the productivities of *all* raw capital in the economy, both his own and others' productivities, i.e. whether  $\Omega_i = \{\{\theta_h\}_{h \in (0,1)}\}$  or  $\Omega_i = \emptyset$ .<sup>1</sup> At stage two the agent commits to work a given amount of hours based on his available information. At stage three, raw capital can be transformed into *intermediate capital* according to a linear production function  $y(x) = (\bar{\theta} + \theta)x$ , where  $\bar{\theta}$  is a deterministic component assumed large enough to guarantee that intermediate capital is positive almost surely.<sup>2</sup> In this stage we will introduce the possibility of trading raw capital, which implies that an agent may produce intermediate capital both with own raw capital and with raw capital purchased

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<sup>1</sup>The stark assumption of no information or perfect information allows us to characterize the main results sharply. We relax the assumption later.

<sup>2</sup>More precisely, such that  $\Pr(\bar{\theta} + \theta_i < 0) \approx 0$ .

from other agents. In the last stage, intermediate capital becomes finally productive once combined with (the already committed) labor, generating *consumption goods*, according to the following production function:

$$C_i = e^{\hat{k}_i(\theta_i)} L_i, \quad (2)$$

where  $\hat{k}_i(\theta_i)$  is the quantity of intermediate capital that agent  $i$  has available after stage three.

While assuming that preferences are characterized by a CRRA utility function (equation 1) is standard in macroeconomics, the assumption that production is characterized by an exponential function (equation 2), and then increasing returns to scale on capital, is not. As it will become clearer, this assumption is extremely convenient for the exposition of the main forces, but not for the main qualitative results.

Now we will go through the different stages backwards. As the last stage is given just by a deterministic production of consumption goods based on equation (2), we first study the potential market for raw capital and production of intermediate capital that take place in the third stage. Then we discuss labor choices during the second stage that are conditional on an information set. The solutions of these stages are enough to study the social optimum level of information in the economy. In the next section, we will dig into the core of our analysis: individual information choices in the first stage and its comparison with social optimum.

## 2.1 Third Stage: Markets for raw capital

In the third stage, raw capital is transformed in intermediate capital. To introduce a market and asset trading in the model, we assume that only a fraction  $\beta$  of the raw capital belonging to an agent  $i$  reveals productive for generating intermediate capital whereas the rest  $1 - \beta$  is useless in the hands of agent  $i$ . This fraction, however, can be sold by agent  $i$  to other agents  $h \neq i$ , who can use it productively to generate intermediate capital. Symmetrically, agent  $i$  can use the proceedings of these sales to buy raw capital that is unproductive in the hands of other agents  $h \neq i$  to use it

in the own production of intermediate capital.

This reallocation of raw capital across agents happens through a market for raw capital, after information and labor has been chosen but before production of consumption goods takes place. As we will show, information that was produced in the initial stage can become pervasive for the operation of this market, in particular increasing the ex-ante volatility of raw capital prices, *a negative face of information*.

What is the trading protocol in the market of raw capital? We assume that agent  $h \neq i$  can buy raw capital from agent  $i$  in a centralized platform that is specific to raw capital  $i$  – let us denote this demand by  $z_h(i)$  – at a unit price  $R_i$ , which represents an enforceable claim on agent  $h$ 's future production of intermediate capital. In other words, agent  $h$  can use raw capital from agent  $i$  that is not productive in agent  $i$ 's hands, and promise to repay with intermediate capital that she will generate. There is, however, a quadratic adjustment cost  $\gamma z_h^2(i)/2$  in units of intermediate capital that agent  $h$  has to incur to operate with other agents' raw capital.

Since labor is already fixed in this stage and production of consumption goods is increasing in the amount of available intermediate capital (from equation 2), in this stage agent  $i$  seeks to maximize the total quantity of intermediate capital to operate. After selling his own raw capita at price  $R_i$ , buying raw capital at a price  $R_h$  from agents  $h \neq i$  and covering adjustment costs, the amount of intermediate capital available to agent  $i$  to produce consumption goods in the last stage is

$$\hat{k}_i(\theta_i) = \beta (\bar{\theta} + \theta_i) + (1 - \beta)R_i + \int \Pi_i(h)dh \quad (3)$$

where  $\Pi_i(h)$  is agent  $i$ 's profit from buying agent  $h$ 's raw capital and given by

$$\Pi_i(h) = (\bar{\theta} + \theta_h) z_i(h) - \frac{\gamma}{2} z_i^2(h) - R_h z_i(h), \quad (4)$$

In words, an agent will operate with intermediate goods that come from three sources. First, by transforming a fraction  $\beta$  of its own raw capital into intermediate capital with productivity  $\theta_i$ . Second, by selling a fraction  $1 - \beta$  of its own raw capital to other agents in exchange for  $R_i$  of intermediate capital from other agents per unit of

own raw capital. Finally, by buying raw capital from other agents and obtaining a profit in terms of intermediate capital production after repayment.

As  $R_i$  and  $\Pi_i(h)$  are endogenous objects, the next Lemma characterizes, as function of primitives, the quantity of intermediate capital available to agent  $i$  after participation in this market.

**Lemma 1.** *The quantity of intermediate capital that agent  $i$  will have available for producing consumption goods is*

$$\hat{k}_i(\theta_i) = \bar{\theta} + \beta\theta_i + (1 - \beta)E_{-i}(\theta_i) + \frac{\Delta_i}{\gamma} - \frac{\gamma}{2}(1 - \beta)^2. \quad (5)$$

where

$$\Delta_i = \int \Delta_{i,h} dh = \text{cov}((\theta_h - E_{-i}(\theta_h))(E_i(\theta_h) - E_{-i}(\theta_h))) - \text{var}((E_i(\theta_h) - E_{-i}(\theta_h)))/2$$

and  $E_{-i}(\theta_h)$  is the average expectation of agents other than  $i$  about the productivity of agent  $h$ 's raw capital. Further,  $\Delta_i = 0$  in case of symmetric information.

*Proof.* First, we obtain the price  $R_i$  that agent  $i$  receives for his own raw capital. Each agent  $h$  chooses the quantity  $z_h(i)$  of raw capital to demand from agent  $i$  to maximize her expected profits, which from agent  $h$ 's version of equation (4) are given by

$$E_h[\Pi_h(i)] = E_h \left[ (\bar{\theta} + \theta_i) z_h(i) - \frac{\gamma}{2} z_h^2(i) - R_i z_h(i) \right].$$

Then, the agent  $h$ 's demand of agent  $i$ 's raw capital is

$$z_h^*(i) = \frac{\bar{\theta} + E_h(\theta_i) - R_i}{\gamma}.$$

Since the supply of agent  $i$ 's raw capital is  $1 - \beta$ , market clearing implies

$$\int z_h^*(i) dh = 1 - \beta,$$

and the equilibrium price in the market of agent  $i$ 's raw capital is

$$R_i = \bar{\theta} + E_{-i}(\theta_i) - \gamma(1 - \beta), \quad (6)$$

where  $E_{-i}(\cdot) = \int_{h \neq i} E_i(\cdot) dh$  is the average expectation across agents other than  $i$  about the productivity of agent  $i$ 's raw capital.

The actual profit of an agent  $h$  as a buyer of agent  $i$ 's raw capital can be then rewritten as

$$\Pi_h(i) = \frac{\Delta_{h,i}}{\gamma} + (\theta_i - E_{-i}(\theta_i))(1 - \beta) + \frac{\gamma}{2}(1 - \beta)^2,$$

where

$$\Delta_{h,i} = (\theta_i - E_{-i}(\theta_i))(E_h(\theta_i) - E_{-i}(\theta_i)) - \frac{1}{2}(E_h(\theta_i) - E_{-i}(\theta_i))^2, \quad (7)$$

depends on the distance between agent  $h$ 's own expectation about  $\theta_i$  and the average expectation of all other agents in the market. No matter the fraction of agents acquiring perfect information, we have that necessarily  $\int E_{-i}(\theta_i) di = \int E_h(\theta_i) di = 0$ . Thus, from the law of large numbers these profits are deterministic so that their ex-ante and ex-post evaluations coincide. In fact, the expected total profits of an agent  $h$  is given by the expectation of actual profits for the raw capital of each agent, then

$$\int E_h(\Pi_h(i)) di = \int \Pi_h(i) di = \frac{\Delta_h}{\gamma} + \frac{\gamma}{2}(1 - \beta)^2, \quad (8)$$

where

$$\Delta_h = cov((\theta_i - E_{-i}(\theta_i))(E_h(\theta_i) - E_{-i}(\theta_i))) - \frac{var((E_h(\theta_i) - E_{-i}(\theta_i)))}{2}, \quad (9)$$

with  $\Delta_h = \int \Delta_{h,i} di$ , denotes the component of profits originating from heterogeneous information: this is zero in case agents hold the same information.

The equation in the lemma comes from substituting the price received from selling raw capital (equation 6) and the profits from buying raw capital (equation 8) symmetrically applied to agent  $i$  from buying raw capital from other agents  $-i$  into

the end of this stage quantity of intermediate capital for agent  $i$  (equation 3).

■

For expositional reasons it will be convenient to decompose the amount of available intermediate capital into a deterministic component,  $\kappa_i$  and a stochastic component,  $k(\theta_i)$ , such that

$$\hat{k}_i(\theta_i) = \kappa_i + k(\theta_i).$$

The deterministic component is

$$\kappa_i = \underbrace{\bar{\theta} - \frac{\gamma}{2}(1 - \beta)^2}_{\kappa} + \frac{\Delta_i}{\gamma}, \quad (10)$$

and the stochastic component is

$$k(\theta_i) = \beta\theta_i + (1 - \beta)E_{-i}(\theta_i), \quad (11)$$

which only depends on the actual productivity of own raw capital (relevant by a fraction of  $\beta$ ) and on other agents' expectation about the productivity of own raw capital (relevant by a fraction of  $1 - \beta$ ). In what follows we denote the *unconditional volatility of the stochastic component* by  $V(k(\theta_i))$ , which can be expressed as

$$V(k_i(\theta_i)) = \beta^2V(\theta_i) + (1 - \beta)^2V(E_{-i}(\theta_i)) + 2\beta(1 - \beta)\text{Cov}(\theta_i, E_{-i}(\theta_i)). \quad (12)$$

**Remark on the nature of trading gains,  $\Delta_i$ :** The solution for  $\Delta_i$  in Lemma 1 depends on own expectations and others' expectations about other agents' raw capital (this is, on  $E_i(\theta_h)$  and  $E_{-i}(\theta_h)$ ). In the Lemma we have been agnostic about the market structure and then how these expectations are formed and depend on such structure. For instance, if agent  $i$  is able to observe the price of raw capital  $R_h$ , knowing that a positive mass  $a$  of agents is informed and there is no noise, he may be able to infer the fundamental value of raw capital, this is  $E_i(\theta_h) = \theta_h$ . Furthermore, agent  $i$  would also understand that other agents are in the same position, and then

$E_{-i}(\theta_h) = \theta_h$ . In this case, which is reminiscent of Grossman and Stiglitz (1980),  $\Delta_i = 0$ , and as long as there are some informed agents in the economy, there are no trading gains in the rational expectations equilibrium. Also as in Grossman and Stiglitz (1980),  $\Delta_i$  is positive only if agent  $i$  is informed and no other agent is, as  $E_i(\theta_h) = \theta_h$  and  $E_{-i}(\theta_h) = 0$ , hence  $\Delta_i = \frac{\text{var}(\theta_h)}{2} = \frac{1}{2}$ .

If, in contrast, we assume that agents do not learn from prices and submit their demand only based on their own expectations, they know that  $E_{-i}(\theta_h) = a\theta_h$ . If agent  $i$  acquires information about  $\theta_h$  (denote it by  $I$ ),  $E_i(\theta_h) = \theta_h$  and  $\Delta_i^I = \frac{1}{2}(1-a)^2$ . If agent  $i$  does not acquire information about  $\theta_h$  (denote it by  $NI$ ),  $E_i(\theta_h) = 0$  and  $\Delta_i^{NI} = -[(1-a)a + \frac{1}{2}a^2]$ . The first expression corresponds to the expected gains from being informed about  $\theta_h$  when a fraction  $a$  of other traders are as well, and declines with  $a$ . The second expression corresponds to the expected costs from not being informed about  $\theta_h$  when a fraction  $a$  of other traders are, and also declines with  $a$ .

Intuitively, the larger the fraction of informed investors in the market, the lower the benefits from being informed, but also the larger the costs of being uninformed. These two effects cancel out as  $a$  changes, such that the net gain from being informed is  $\Delta_i^I - \Delta_i^{NI} = 1/2$ , independent on the fraction of other agents that are informed.

This discussion highlights that expected trading gains depend critically on how expectations are formed. In one extreme, when agents learn perfectly from prices, information gains from trading purposes only exist when no other agent is informed. In the other extreme, when agents do not learn from prices, information gains from trading purposes always exist.

**Remark on selling only unproductive raw capital:** In our setting agents sell all their unproductive raw capital and none of their productive raw capital. On the one hand, an agent does not want to keep any fraction of unproductive raw capital as it can be exchanged in the market for at least some intermediate capital. On the other hand, an agent does not want to sell any own productive raw capital because we have assumed that the agent puts the raw capital in the market before observing its price,  $R_i$ . Then the selling decision is determined by  $E_i(R_i) = \bar{\theta} + E_i(E_{-i}(\theta_i)) - \gamma(1 - \beta)$

(from equation 6), where  $E_i(E_{-i}(\theta_i)) = E_i(\theta_i)$  by the martingale property. If the agent works with the own productive raw capital it obtains in expectation  $\bar{\theta} + E_i(\theta_i)$  of intermediate capital. If the agent sells an unit of raw capital it gets in expectation  $E_i(R_i) = \bar{\theta} + E_i(\theta_i) - \gamma(1 - \beta)$ . The agent then clearly prefers to use as much own raw capital as possible instead of selling it at a discount given by the adjustment cost  $\gamma$  that other agents have to face when using it.

## 2.2 Second Stage: Labor choice

In the second stage, agent  $i$  chooses the amount of labor to commit using in the production of consumption goods during the last stage. To make this choice, however, the agent needs to form an expectation of the quantity of intermediate capital he will have available to combine with labor during the last stage, which was shown before to only depend on the productivity of the own raw capital. Obtaining information about raw capital productivities allows agents to form those expectations more accurately and make better labor decisions, *the positive face of information*.

The next Lemma shows the amount of labor that agent  $i$  commits to combine with his expected (conditional on available information) quantity of intermediate capital to produce consumption goods.

**Lemma 2.** *Conditional on his information set  $\Omega_i$ , agent  $i$  wants to commit working*

$$L_i = E_i[e^{(1-\sigma)(\hat{k}_i(\theta_i))}]^{\frac{1}{\sigma}}. \quad (13)$$

*Proof.* This results follows from maximizing equation (1) subject to (2). ■

Note that labor is increasing in expected intermediate capital when  $\sigma < 1$ , and decreasing when  $\sigma > 1$ . On the one hand, when  $\sigma < 1$  more expected intermediate capital induces more labor because the substitution effect prevails: the more productive is labor (because of the higher level of intermediate capital to operate with) the agent is willing to enjoy less leisure and work more. On the other hand, when  $\sigma > 1$  the opposite occurs as a wealth effect prevails: the more productive is labor, the agent produces more and prefers to cut on labor and enjoy more leisure.

In the next section we compare the economy without information with an economy with costless and perfect information. We show under what conditions a planner prefers no information than perfect information, even though costless. We also show that individuals would always prefer to operate with costless and perfect information given that they do not internalize the role of information on the volatility of markets. In essence we will discuss how a planner would evaluate the positive and negative faces of information, while individuals tend to only focus on the positive face.

### 3 (In)efficiency of Free and Perfect Information

Given the ex-ante symmetry across agents (previous to possibly acquiring signals heterogeneously), the ex-ante utility of an individual is also the utility of a representative agent and coincides with social welfare from an ex-ante perspective.

To disentangle the two faces of information on the individual and social gains of information in the economy, we first discuss a case in which labor is fixed, potentially muting the allocative positive face of information and focusing on its negative face. Then we add this labor choice element and study the trade-off that arises from these two faces operating simultaneously.

#### 3.1 No Labor Choice

If the household cannot choose labor, say because labor supply is fixed at  $\bar{L}$ , then the expected utility of the agent from equation (1) can be written simply as

$$E(U_i) = \frac{\bar{L}^{1-\sigma}}{1-\sigma} e^{(1-\sigma)\kappa_i + \frac{(1-\sigma)^2}{2} V(k(\theta_i))} - \bar{L}$$

by using equation (2) and the decomposition of equation (5) into the deterministic component  $\kappa_i$  and the stochastic component  $k(\theta_i)$  with unconditional variance  $V(k(\theta_i))$  from equation (12).

**Planner's Problem.** Consider a social planner who can decide between acquiring information and provide it to all agents or not acquiring information and prevent

all agents from having it. In this case  $\Delta_i$  is deterministically equal to zero, as information (or lack thereof) is symmetric. The collective use or neglect of information, however, alters the unconditional variance of  $k(\theta_i)$ .

As the expected utility of a representative agent only depends on the unconditional variance of  $k(\theta_i)$ , increasing when  $\sigma < 1$  and decreasing when  $\sigma > 1$ , we can write the social welfare criterion for a given  $\sigma$  as

**Definition 1.** *Without labor choices, for given  $\beta$  and  $\sigma$  social welfare is increasing whenever*

$$\tilde{V}(\theta_i) \equiv \text{sign}(1 - \sigma)V(k(\theta_i)).$$

*is increasing.*

Since we have assumed that  $V(\theta_i) = 1$ , it is easy to see from equation (12) that, when all agents have information,  $V(k(\theta_i)) = 1$ , whereas when no agent has information,  $V(k(\theta_i)) = \beta^2$ .

The unconditional variance of  $k(\theta_i)$  is lower without information because *agents can use market trading to insure against productivity uncertainty*. With perfect information, the uncertainty the agent faces in terms of the amount of intermediate capital at the time of producing consumption goods is determined by the variance of the own raw capital, regardless whether it is used or traded. In contrast, without information, the individual uncertainty the agent faces is the uninsurable part of its raw capital (the fraction  $\beta$  that is not traded), while the unproductive fraction of raw capital is traded in exchange of raw capital of ex-ante deterministic productivity. We have the following proposition as a direct consequence of our welfare criterion.

**Proposition 1.** *From a social point of view, when  $\beta < 1$  information acquisition is beneficial when  $\sigma < 1$  whereas is detrimental when  $\sigma > 1$ .*

What is the role of  $\sigma$  for this result? More specifically, why does utility decreases with variance when  $\sigma > 1$  and decreases when  $\sigma < 1$ ? On the one hand, the utility function is CRRA with respect to consumption, with relative coefficient of *risk-aversion to consumption* equal to  $\sigma$ , and then decreasing in variance for all  $\sigma > 0$ . On the other hand, the production function displays increasing returns of

intermediate capital (this is, there are complementarities across intermediate capital units). While the utility is always decreasing in the variance of consumption (a concave function), consumption is always increasing in the variance of intermediate capital (a convex function). When  $\sigma$  is small enough (in particular less than 1) the second effect dominates and utility increases in the variance of intermediate capital. When  $\sigma$  is large enough (in particular more than 1) the first effect dominates and utility is decreasing in the variance of intermediate capital.

In words, when  $\sigma < 1$ , the effect of the intermediate capital variance in increasing expected consumption (via the exponential production function) is stronger than the effect of the variance in reducing the utility of such consumption. More formally, the relative coefficient of *risk-aversion to intermediate capital*,  $-\frac{U''(\widehat{k})}{U'(\widehat{k})}C = -(1-\sigma)C$ , this is risk-averse with respect to intermediate capital for  $\sigma > 1$  and risk-lover for  $\sigma < 1$ . The critical level of this switch is  $\sigma = 1$  because at such level the utility function is logarithmic in consumption and consumption is exponential on intermediate capital, and then individuals are effectively risk neutral with respect to intermediate capital.

The planner then prefers households to face a higher variance of intermediate capital when  $\sigma < 1$ , as the gains from more consumption surpasses the costs of its volatility, and as such prefers to provide them with perfect information. When  $\sigma > 1$ , the planner would rather households not to have information that would increase the variance in intermediate capital, as the gains from more consumption do not justify the costs in terms of its volatility.

What is the role of  $\beta$ ? When  $\beta = 1$  there is no market for intermediate capital. In this case information is irrelevant as it just represents a useless early resolution of uncertainty. Without labor choices, individuals are not able to use information to change labor optimally, and such information does not affect the variance of intermediate capital that is solely given by the variance of the own raw capital productivity. This leads us to the next corollary.

**Corollary 1.** *When there are no labor choices and there is no market for raw capital (this is,  $\beta = 1$ ) information is irrelevant and does not affect social welfare.*

This section highlights an important benchmark in the absence of labor choices.

When individuals are risk averse (in the appropriate sense of risk aversion vis-a-vis the volatility of production inputs) information is detrimental from a social standpoint, as it reduces the possibilities of insurance that trading provides against idiosyncratic shocks in production. When there is no trade, naturally, the role of information is moot. This benchmark then, highlights a purely *negative face of information*.

**Individual's Problem.** Consider now how individuals value information. From equation (12) it is clear that agent  $i$ 's own information does not affect the volatility component of her own unconditional expected utility, as  $V(\theta_i)$  is the unconditional volatility of  $\theta_i$ , and  $V(E_{-i}(\theta_i))$  depends on the information acquisition of others. The only component that is affected by the individual choice of acquiring information is  $\Delta_i$ , which is an element of  $\kappa_i$ . Intuitively, as agents do not have any labor choice, information becomes relevant only for the agent's potential advantage on trading raw capital. As we discussed above, these gains depend on how expectations are formed and then how informative prices are. Regardless, as long as there is a small chance to being the only informed agent, and as information is assumed free and perfect, individuals always find optimal to have access to such information.

This result highlights the source of externality in our setting. Individuals never internalize the social value of information in increasing the unconditional variance of  $k(\theta_i)$ . When  $\sigma < 1$  this effect increases average consumption more than its volatility (in terms of utility), being socially beneficial such that the externality does not generate a misalignment between the social optimum and individual choices. When  $\sigma > 1$ , however, the increase in the unconditional variance of intermediate capital increases average consumption less than its volatility (in terms of utility) generating a misalignment between social optimum and individual choices. The fact that the outcome of the externality only bites when  $\sigma > 1$  is just a result of the bang-bang solution of either having full or no information, not that the externality only exists in a range of parameters, as this externality always exist.

Finally, even though these results are based on a highly streamlined and simplified setting (information is costless and perfect, the unconditional distribution of  $k(\theta_i)$  is not mean invariant and the production function is exponential) we will discuss later

how the main insights extend to more general environments.

### 3.2 Labor Choice

Now we allow for labor choices, which introduces a positive allocative face of information. The ex-ante expected utility of an agent (and the social welfare) is

$$E[U_i] = E \left[ \frac{e^{(1-\sigma)\hat{k}_i(\theta_i)} E_i[e^{(1-\sigma)\hat{k}_i(\theta_i)}]^{1-\sigma}}{1-\sigma} - E_i[e^{(1-\sigma)\hat{k}_i(\theta_i)}]^{1-\sigma} \right],$$

which obtains after replacing the optimal choice of labor (13) and the production function (2) into the utility (1), where  $E[\cdot]$  denotes the unconditional expectation operator.

As before, we decompose  $\hat{k}(\theta_i)$  from equation (5) into a deterministic component  $\kappa_i$  and a stochastic component  $k(\theta_i)$ . Given the properties of log-normal distributions (see full derivation in the Appendix 1) we have

$$E[U_i] = \frac{\sigma}{1-\sigma} e^{\frac{(1-\sigma)^2}{2\sigma} \left( \frac{1}{\sigma} \underbrace{V(E_i(k(\theta_i)))}_{V(k(\theta_i)) - V_i(k(\theta_i))} + V_i(k(\theta_i)) + \frac{2}{1-\sigma} \kappa_i \right)}, \quad (14)$$

which depends, as before, on the deterministic component  $\kappa_i$  and the unconditional variance of  $k(\theta_i)$ , but now also on the the variance of  $k(\theta_i)$  conditional to the information set held by agent  $i$  (this is, her forecast error) that we distinguish from the unconditional variance by denoting it as  $V_i(k(\theta_i))$ .

The interpretation of how labor choices affect expected utility is intuitive. When information is irrelevant, such that the variance the individual faces with information is the same as without information (this is  $V(k(\theta_i)) = V_i(k(\theta_i))$ ), then the variance affects expected utility more than when information is perfect and there are no forecast errors (this is,  $V_i(k(\theta_i)) = 0$ ), in which case the variance affects utility scaled by  $1/\sigma$ , which measures the intensity to which labor reacts to information about the variance. When  $\sigma > 1$  labor moves in opposite direction to news about  $\hat{k}$ , dampening the effect of the intermediate capital variance. When  $\sigma < 1$  labor moves in the same direction

as news about  $\hat{k}$ , strengthening the effect of the intermediate capital variance. As utility decreases with variance of intermediate capital when  $\sigma > 1$  and increases when  $\sigma < 1$ , in both cases being able to adjust labor lead to an improvement of expected utility.

**Planner's Problem.** As before, we take the point of view of a social planner who can decide on either all agents have information or none has. Also as before  $\Delta_i$  is deterministically equal to zero. In this setting, however, the collective use or neglect of information not only alters the unconditional variance of  $k(\theta_i)$  but also the variance of  $k(\theta_i)$  conditional to the information set of agent  $i$ , i.e. her forecast error variance. This is clear in the adjusted welfare criterion below.

**Definition 2.** *With labor choices, for given  $\beta$  and  $\sigma$  social welfare is increasing whenever*

$$\mathbf{V}(\theta_i) \equiv \text{sign}(1 - \sigma) \left( \frac{1}{\sigma} V(k(\theta_i)) - \frac{1 - \sigma}{\sigma} V_i(k(\theta_i)) \right)$$

*is increasing.*

As in the case without labor choices, the unconditional volatility of  $k(\theta_i)$ , with  $\sigma < 1$  this term increases welfare and when  $\sigma > 1$  this term decreases welfare. There is, however, an additional variance component that determines welfare when agents can exploit information ex-ante for labor choices, this is the variance that comes from making mistakes once labor has been chosen, which is captured by  $V_i(\cdot)$  and represents the variance of the intermediate capital available to work after fixing labor. This extra benefit of information bends the previous result that information is never optimal when  $\sigma > 1$  and indeed qualifies this result, making information beneficial for a range of parameters when  $\sigma > 1$ . The next proposition shows that in our setting we can capture this trade-off in a very tractable and intuitive way.

**Proposition 2.** *Information acquisition is socially desirable when*

$$\sigma < 1 \quad \text{or} \quad \sigma > \frac{1}{\beta^2}$$

*and socially detrimental otherwise.*

*Proof.* Without information agents' forecasts about the own raw capital productivity are equal to the prior mean ( $E_i(\theta_i) = 0$ ), so  $k(\theta_i) = \beta\theta_i$  and the forecast error variance is equal to the unconditional variance, i.e.  $V_i(k(\theta_i)) = V(k(\theta_i)) = \beta^2V(\theta_i) = \beta^2$ , so that  $\mathbf{V}(\theta_i) = \text{sign}(1 - \sigma)\beta^2$ . With full information, agents' forecasts are equal to the realization of  $\theta_i$ , so  $k(\theta_i) = \theta_i$ , the unconditional variance is  $V(k(\theta_i)) = 1$  and the forecast error variance is  $V_i(k(\theta_i)) = 0$ , so that  $\mathbf{V}(\theta_i) = \text{sign}(1 - \sigma)\frac{1}{\sigma}$ . The Proposition follows from comparing these two values of the welfare criteria. ■

Intuitively, the trade-off between information (costless and perfect) and no information then gets determined by the trade-off between *insurance by reallocating labor* and *insurance by trading raw capital*. Perfect information provides insurance by reallocation through reducing the forecast error (this is, the conditional variance). By providing information the agent can react to the uncertainty about productivities by choosing labor more efficiently: *agents can use labor choices to insure against uncertainty*. This source of self-insurance is achieved by providing more information and becomes relatively more valuable as agents become more risk averse (higher  $\sigma$ ).

In contrast, as discussed in the case of no labor choices, no information provides insurance by trading raw capital through reducing the unconditional variance. This source of insurance is achieved by avoiding information and becomes relatively more valuable as trading becomes more prevalent (lower  $\beta$ ).

**Individual's Problem.** In the expression for expected utility, there are now two components that are affected by the individual choice of acquiring information, namely the volatility of  $k(\theta_i)$  conditional to the information set held by agent  $i$ ,  $V_i(k(\theta_i))$ , and  $\Delta_i$ . The former captures utility gains originating from the possibility to adjust labor in response to information that reduces the uncertainty about the amount of intermediate capital that the agent will have available to produce. The later, as in the case without labor choices, improves the individual position of the agent when trading raw capital. This implies that information both minimizes the forecast error variance and maximize returns from trading, for any value of  $\sigma$ , reinforcing the individual incentives to acquire information. Since in our benchmark without labor choices individuals would always prefer to have costless and perfect

information, these incentives are further reinforced for all parameter values when labor can adjust.

When  $\sigma < 1$ , individuals not only fail internalizing the social value that information has in increasing the unconditional volatility of  $k(\theta_i)$  but also choose to work more when intermediate capital is high as this would boost the expected consumption. In other words, agents take into account the gains of information in reducing the forecast error but not the gains that come from his own reaction to increase the unconditional variance of consumption. Again, when  $\sigma < 1$ , although there is a discrepancy between individual and social evaluation of information acquisition both the planner and the individuals prefer full information to no information. Even though their incentives are not perfectly aligned the outcomes are, given the corner solutions of full information.

When  $\sigma > 1$  individuals always want to acquire information but the planner faces a trade-off. On the one hand, information deteriorates insurance by trading. Without information agents cannot allocate labor optimally for the fraction of productive own raw capital but they do not face uncertainty for the fraction of raw capital that is traded (a deterministic amount when there is no information about the own raw capital) – *agents can use trading to insure against productivity uncertainty*. This insurance gets relatively more valuable as agents have larger trading need (lower  $\beta$ ). On the other hand, information provides insurance by labor allocation. With information agents face uncertainty for the fraction of raw capital that is traded but can allocate labor optimally for the fraction that is not – *agents can use labor allocation to insure against productivity uncertainty*. This insurance gets relatively more valuable as agents have a higher risk aversion (higher  $\sigma$ ). Therefore, with labor choices, the planner does not always provide information when  $\sigma > 1$ , but only when  $\frac{1}{\sigma} < \beta^2$ , or when  $\sigma > \frac{1}{\beta^2}$ . This implies that, again because of the corner solution, when  $1 < \sigma < \frac{1}{\beta^2}$  the misalignment of incentives between the planner and the individuals lead also to a misalignment of outcomes.

### 3.3 Discussions

**Remark on the limit of  $\sigma \rightarrow \infty$ :** To better appreciate why information has a high social value for sufficiently high values of risk aversion, it is useful to look at the limit case of  $\sigma \rightarrow \infty$ . At this limit, the social welfare function is simply given by

$$\lim_{\sigma \rightarrow \infty} \mathbf{V}(\theta_i) = -V_i(k(\theta_i)).$$

This is intuitive looking at the log-linearized version of (13), namely  $l_i = E_i[\hat{k}(\theta_i)](1 - \sigma)/\sigma$ , that plugged into log-linearized consumption gives  $c_i = \hat{k}(\theta_i) + l_i$  or at the limit

$$\lim_{\sigma \rightarrow \infty} c_i = \hat{k}(\theta_i) - E_{i,j}[\hat{k}(\theta_i)].$$

Therefore, when aversion to consumption volatility is extreme, consumption is equal to the individual forecast error. Under perfect information, the agent works exactly to offset the effect of fluctuations in productivity on individual consumption, or in other words, to minimize forecast error variance: this is the result of an extremely powerful wealth effect. As a consequence, providing agents with information in such conditions is clearly optimal from a social perspective.

**Remark on the empirical plausibility of socially inferior information:**

Our setting provides a simple macro-finance framework that allows us to determine the social desirability of costless perfect information just comparing two parameters that have empirical counterparts.

When is information socially undesirable? Information is not desirable when risk aversion is not so high relative to trading needs, in particular when  $\sigma < \frac{1}{\beta^2}$ . How do these parameters look like in reality? There is a wide consensus on most calibrations of macroeconomic models that rely on CRRA utility functions in using  $\sigma = 2$ . This implies that free and perfect information is socially undesirable as long as  $\beta < \sqrt{0.5} = 0.71$ .

There is less consensus about the fraction of trading capital on total production. The parameter  $1 - \beta$ , however, would roughly corresponds to the fraction of intermediate input used in production. In other words,  $1 - \beta$  would correspond to the

fraction of inputs bought from other firms or by the fraction of own production sold to other firms to be used by them as their intermediate inputs. Jones (2010) estimate the share of intermediate inputs in total production for 35 countries in 2000, and find that this number ranges between 0.41 in Greece and 0.68 in China, with an average of 0.53 and a standard deviation of 0.06. This implies that empirically  $\beta \in [0.32, 0.59]$ , well below the conservative magnitude needed for perfect and costless information to be socially desirable in our model.<sup>3</sup>

This is likely a conservative condition because we have obtained it by assuming (i) increasing returns in the production of consumption goods (more precisely exponential on intermediate capital) and (ii) costless and perfect information, both empirically implausible. First, increasing returns to scale increases the benefits of information because a higher unconditional variance of inputs generates a higher average consumption. This implies that with more plausible production functions with decreasing returns to scale (or weaker increasing returns) this effect shrinks and it is more likely for perfect and costless information to be socially undesirable, as we generalize next. Second, perfect and costless information also increases the benefits of information. Assuming that information is imprecise and there is cost in generating it also reduces the chances for information production to be a socially desirable activity.

**Remark on the role of incomplete markets:** The detrimental effect of information that we document here critically relies on (i) the lack of other ways to insure against consumption risks and (ii) the lack of systemic risk.

With respect to (i), if households have a way to insure against consumption risk through contracts that can be implemented and enforced, then the role of information in increasing the volatility of market prices and then increasing consumption risks becomes irrelevant. In this sense, our conclusions rely on the existence of incomplete markets.

With respect to (ii), assume perfectly correlated productivity across agents' raw

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<sup>3</sup>These numbers are consistent with previous estimations by Basu (1995) of  $\beta = 0.5$  (based on the U.S. between 1947 and 1979) and Ciccone (2002) of  $\beta = 0.3$  (based on the experiences of South Korea, Taiwan and Japan in the seventies).

capital, this is  $\theta_i = \theta$  for all  $i$ . In this case, the amount of intermediate capital available to produce consumption goods by agent  $i$  (following equation 5) is

$$\hat{k}_i(\theta_i) = \bar{\theta} + \beta\theta + (1 - \beta)E_{-i}(\theta) + \frac{\Delta_i}{\gamma} - \frac{\gamma}{2}(1 - \beta)^2. \quad (15)$$

In the case of symmetric information (e.g. everybody knows  $\theta$  or nobody does) this expression simply becomes

$$\hat{k}_i(\theta) = \bar{\theta} + \theta - \frac{\gamma}{2}(1 - \beta)^2 \quad (16)$$

that is, the dependence from expectations vanishes because, as the risk becomes uninsurable, the gain (or loss) of an household in his role of trader will be exactly offset by the loss (or gain) in the role of consumer-worker. Thus, it is crucial that in our original economy a household is perfectly insured through portfolio diversification against the risk she faces in trading, but she has no other instrument that her own labor choice or the ignorance of the market to insure against consumption risk.

### 3.4 Generalizing the results

The previous results are based on a set of specific functional forms that we adopted for tractability and for clarity in the exposition. In particular

1. The production function of consumption goods is special (exponential on intermediate capital). The special implication is that individuals are risk neutral on intermediate capital goods when their preferences are logarithmic in consumption goods.
2. The production function of intermediate goods is also special (linear in the productivity of raw capital).
3. The combination of these two assumptions imply that the unconditional distribution of  $e^{k(\theta)}$  is not mean invariant because the expectation of production is not the same as the production of the expectation, this is,  $E(e^{k(\theta)}) =$

$$e^{E(k(\theta)) + \frac{V(k(\theta))}{2}} \neq e^{E(k(\theta))}.$$

In this section we consider a generic form of production functions, both of consumption goods and intermediate goods, such that the unconditional distribution of consumption goods is mean invariant. To be more precise, now we assume a general production function for consumption goods

$$C_i = F(k(\theta))L_i,$$

and a general production function for intermediate capital goods

$$k(\theta) = f(\beta\theta + (1 - \beta)E[\theta|\Omega]).$$

We assume that  $f(\cdot)$  is such that, without information ( $\Omega = \emptyset$ )  $E[F(k(\theta))] = F(E[\theta])$ ; this is the unconditional distribution of  $F(k(\theta))$  is mean invariant, which without loss of generality we fix to be equal to 1. For technical reasons we also assume  $L_i \in [0, L^+]$  and  $L^+$  arbitrarily large but bounded. These generalizations also imply that without information  $L_i = \bar{L} = E[k(\theta)^{1-\sigma}]^{\frac{1}{\sigma}}$  and with information  $L_i = L(\theta) = k(\theta)^{\frac{1-\sigma}{\sigma}}$ .

With this generalization we show that the main results that costless and perfect information is not socially desirable when  $\beta$  is sufficiently small and/or when  $\sigma$  lies in an intermediate range are not a special result from special assumptions but indeed robust to a generalization of the production function. We do this in parts. First we show that information is always preferred in the limits  $\sigma \rightarrow 0$  and  $\sigma \rightarrow \infty$  and never preferred when  $\sigma = 1$  provided that the production function for consumptions goods is less convex than exponential. Second we show that information is always preferred when  $\beta = 1$  (there is no market for raw capital) and when  $\beta = 0$  provided  $\sigma < 1/2$ .

### 3.4.1 The limit $\sigma \rightarrow 0$ .

In proposition 2 we have shown with simplified assumptions that information is always preferred when  $\sigma < 1$ . Here we show that, regardless of the production function for consumption and intermediate capital goods, it is always the case that

information is preferred when  $\sigma$  is small enough. The utility function when  $\sigma \rightarrow 0$  can be approximated by

$$\lim_{\sigma \rightarrow 0} E(U_i) = E[(F(k(\theta)) - 1)L_i],$$

and it is straightforward to show that

$$E[(F(k(\theta)) - 1)\bar{L}] = E[(F(k(\theta)) - 1)]\bar{L} < E[(F(k(\theta)) - 1)L(\theta)] = \quad (17)$$

$$= E[(F(k(\theta)) - 1)]E[L(\theta)] + cov(L(\theta), F(k(\theta))) \quad (18)$$

as given rationality,  $cov(L(\theta), F(k(\theta)))$  is positive.

When  $\sigma \rightarrow 0$ , consumption volatility has negligible individual and social costs, then agents (and also the planner) want to maximize the expected level of consumption. This is achieved setting  $L_i = L^+$  whenever  $F(k(\theta)) \geq 1$  and  $L_i = 0$  otherwise. Intuitively agents have incentive to exploit all the flexibility in their allocation choice and the planner wants them to have the information to react upon.

### 3.4.2 The limit $\sigma \rightarrow \infty$ ?

According to proposition 2 information is always preferred when  $\sigma \rightarrow \infty$ . As discussed in the remark on this limit, this is also the case when generalizing production functions as the welfare function still depends purely on the forecast error, and this is minimized by acquiring information.

### 3.4.3 The case of $\sigma = 1$

In proposition 2 we have shown that the social planner is indifferent between having information or not when  $\sigma = 1$ . This particular result is the knife edge unique implication of our simplifying assumption, as the individuals are risk neutral with respect to intermediate capital goods exactly when  $\sigma = 1$ . More generally, however, when the production function is general and the unconditional distribution of consumption goods is mean invariant the planner does not want information when  $\sigma = 1$  if  $F(k(\theta))$

is less convex than exponential and wants information when  $\sigma = 1$  if  $F(k(\theta))$  is more convex than exponential.

This result arises from recognizing that when  $\sigma = 1$  labor is optimally given by  $L_i = 1$ , which is equivalent to the situation without labor choice. If labor is fixed then information only matters as it moves the variance of  $F(k(\theta))$ . We have

$$E[U_i] = E[\log F(k(\theta)) - 1],$$

This function is a decreasing function of the volatility of  $k(\theta)$  if  $\log F(k(\theta))$  is concave and an increasing function of the volatility of  $k(\theta)$  if  $\log F(k(\theta))$  is convex. An immediate implication is that the no information regime is always socially preferred when  $\sigma = 1$  and the production function of consumption goods displays decreasing returns on the amount of intermediate capital.

#### 3.4.4 The case $\beta = 1$ .

In proposition 2 we have shown that, when there is no market for raw capital, information is strictly preferred for any  $\sigma$  (and weakly so when  $\sigma = 1$ ). Under more general assumptions this result remains. In this case the variance of  $F(k(\theta))$  is fixed irrespective of the information regime and then we can treat it as exogenous. With perfect information we have

$$E[U_i] = E\left[\frac{F(k(\theta))^{\frac{1-\sigma}{\sigma}}}{1-\sigma} - F(k(\theta))^{\frac{1-\sigma}{\sigma}}\right] = \frac{\sigma}{1-\sigma} E\left[F(k(\theta))^{\frac{1-\sigma}{\sigma}}\right].$$

With no information we have

$$E[U_i] = \frac{E[F(k(\theta))^{1-\sigma}]E[F(k(\theta))^{1-\sigma}]^{\frac{1-\sigma}{\sigma}}}{1-\sigma} - E[F(k(\theta))^{1-\sigma}]^{\frac{1}{\sigma}} = \frac{\sigma}{1-\sigma} E[F(k(\theta))^{1-\sigma}]^{\frac{1}{\sigma}} \quad (19)$$

Hence, the perfect information scenario is preferred whenever

$$\frac{\sigma}{1-\sigma} E\left[F(k(\theta))^{\frac{1-\sigma}{\sigma}}\right] > \frac{\sigma}{1-\sigma} E[F(k(\theta))^{1-\sigma}]^{\frac{1}{\sigma}}$$

or

$$\begin{aligned} E \left[ F(k(\theta))^{\frac{1-\sigma}{\sigma}} \right] &> E[k(\theta)^{1-\sigma}]^{\frac{1}{\sigma}} \quad \text{with } \sigma < 1 \\ E \left[ F(k(\theta))^{\frac{1-\sigma}{\sigma}} \right] &< E[F(k(\theta))^{1-\sigma}]^{\frac{1}{\sigma}} \quad \text{with } \sigma > 1 \end{aligned}$$

which is always true because of the Jansen inequality since  $y(x) = x^{\frac{1}{\sigma}}$  is convex with  $\sigma < 1$  and concave otherwise (consider  $x = k(\theta)^{1-\sigma}$  as an arbitrary random variable).

Intuitively, when there is no market and then no externality of individual informational choices in affecting the utility of other individuals by affecting the market volatility (as there is no market), informational choices are always socially optimal and agents always want to acquire information.

### 3.4.5 The case $\beta = 0$ .

In proposition 2 we have shown that information is never preferred for  $\sigma > 1$  when  $\beta = 0$ . In this case, the variance of  $F(k(\theta))$  is zero without information, in particular, under the more general specification,  $F(k(\theta)) = F(E[\theta])$  for any realization of  $\theta$ . In this case, information is socially preferred whenever

$$\frac{\sigma}{1-\sigma} E \left[ F(k(\theta))^{\frac{1-\sigma}{\sigma}} \right] > \frac{\sigma}{1-\sigma} E[F(E[\theta])^{1-\sigma}]^{\frac{1}{\sigma}} = \frac{\sigma}{1-\sigma} E[F(k(\theta))]^{\frac{1-\sigma}{\sigma}}$$

or

$$\begin{aligned} E \left[ F(k(\theta))^{\frac{1-\sigma}{\sigma}} \right] &> E[F(k(\theta))]^{\frac{1-\sigma}{\sigma}} \quad \text{with } \sigma < 1 \\ E \left[ F(k(\theta))^{\frac{1-\sigma}{\sigma}} \right] &< E[F(k(\theta))]^{\frac{1-\sigma}{\sigma}} \quad \text{with } \sigma > 1. \end{aligned}$$

Because of the Jensen inequality information is socially preferred only for  $0 < \sigma < 1/2$ , whereas no information is preferred otherwise. This result obtains from the fact that the function  $x^{\frac{1-\sigma}{\sigma}}$  is concave only in the range  $\sigma \in (1/2, 1)$ .

More precisely, with  $\sigma \in (0, 1/2)$  labor expansions are more sizable on average than labor contractions so volatility in productivity is socially desirable. On the contrary, with  $\sigma > 1/2$  labor expansions are less sizable on average than labor

contractions; volatility in productivity is then socially inferior. In this second case, given that individuals do not internalize the effect of information acquisition on the volatility of productivity, the social planner prefers not releasing any information to agents.

In our benchmark case, this threshold at  $\sigma = 1/2$  is not visible because, contrarily to our general specification, the average productivity increases with the variance of productivity. This is why the threshold is higher (at  $\sigma = 1$ ) than in the general case.

### 3.4.6 An example

Here we provide an example that is slightly different than our original and simplified setting to show how the range in which information is socially inferior changes with generalizations. We assume that the production function of consumption goods is still exponential, while the production function for intermediate capital is still linear, but adjusted such that the unconditional distribution of consumption goods is mean invariant. More specifically, we assume that

$$F(\cdot) = e^{(\cdot)} \quad \text{and} \quad k(\theta) = \beta\theta + (1 - \beta)E[\theta|\Omega] - \frac{1}{2}V(k(\theta)).$$

The additional term  $-\frac{1}{2}V(k(\theta))$  corrects for the effect of a change in unconditional variance on the unconditional mean, so that  $E[F(k(\theta))] = F(E[\theta])$  in case of no information. This additional term is deterministic and therefore can be easily included in our analysis as an element in the deterministic component,  $\kappa_i$ .

Without labor choices the criterion reads as

$$\begin{aligned} E(U_i) &= \frac{\bar{L}^{1-\sigma}}{1-\sigma} e^{(1-\sigma)(\kappa_i - \frac{1}{2}V(k(\theta))) + \frac{(1-\sigma)^2}{2}V(k(\theta_i))} - \bar{L} \\ &= E(U_i) = \frac{\bar{L}^{1-\sigma}}{1-\sigma} e^{(1-\sigma)\kappa_i - \frac{\sigma(1-\sigma)}{2}V(k(\theta_i))} - \bar{L}, \end{aligned}$$

which shows that without labor choice no information is always socially preferred to information.

With labor choice, the new expected utility function will read as

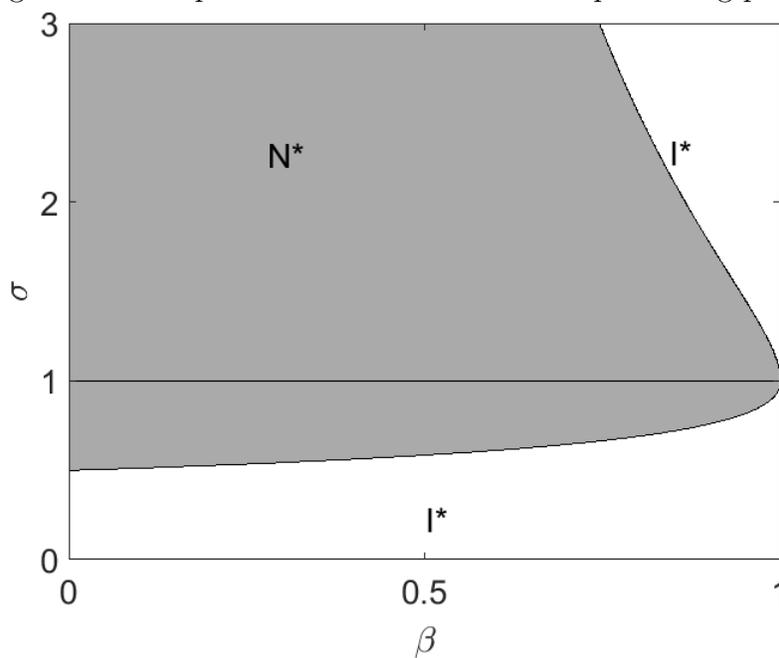
$$E[U_i] = \frac{\sigma}{1-\sigma} e^{\frac{(1-\sigma)^2}{2\sigma} \left( \frac{1}{\sigma} V(k(\theta_i)) + \left( \frac{\sigma-1}{\sigma} \right) V_i(k(\theta_i)) \right) + \frac{1-\sigma}{\sigma} \left( \kappa_i - \frac{1}{2} V(k(\theta)) \right)},$$

so that the new criterion obtains as

$$\begin{aligned} \mathbf{V}(\theta_i) &\equiv \text{sign}(1-\sigma) \left( \frac{1}{\sigma} V(k(\theta_i)) - \frac{1-\sigma}{\sigma} V_i(k(\theta_i)) - \frac{1}{1-\sigma} V(k(\theta)) \right) \\ &= \text{sign}(1-\sigma) \left( \frac{1-2\sigma}{\sigma(1-\sigma)} V(k(\theta_i)) - \frac{1-\sigma}{\sigma} V_i(k(\theta_i)) \right). \end{aligned}$$

This case is depicted in Figure 1 and confirms the findings of our general analysis. The white area displays the region where information is socially beneficial while the

Figure 1: Example with unconditional mean preserving profit.



grey area displays the region where no information is optimal. The absence of any positive effect of volatility on mean, which is proper of our benchmark setting, reduces

the region where information is socially preferred.

Intuitively, the correction to maintain mean invariance implies that more variance of  $\theta$  does not translate into a higher expected level of consumption goods, even though the production function is exponential. This positive effect of the variance then reduces the range for which information is optimal.

Notice also that, consistent with our generalization results information is always optimal if  $\beta = 1$  and is optimal when  $\beta = 0$  only if  $\sigma < 1/2$ . Similarly, when  $\sigma$  is small or large enough, information is optimal regardless of  $\beta$  and, given the correction that affects the expected production of consumption goods, in this case information is never optimal when  $\sigma = 1$ .

In the next two sections we further generalize our insights by assuming, (i) that information is imperfect and (ii) that it is costly to obtain.

## 4 Imperfect Information

To better highlight the trade-off that the social planner faces we relax now the assumption that information is perfectly precise. We assume that the social planner can control the precision of information that an agent can have about the productivity of her own raw capital and the precision of information that an agent can have about others' raw capital productivity.

This flexibility allows us to highlight two new results. First, in terms of optimal provision of information, (i) the planner's preferred plan of action would be providing perfect information about own raw capital but no information about others' raw capital and (ii) for a range of parameters no information is preferred to imprecise information. Second, even though perfect information is always optimal when  $\sigma \rightarrow \infty$ , this is not necessarily the case when information is not perfect.

We assume that the social planner can release a *private* Gaussian signal about the realization of each agent's own capital productivity, with precision  $\tau_i$ . Just before the trading stage, and after agents fixed their supply of labor, the social planner can also release a *public* Gaussian signal about the productivity of each type of raw

capital, with precision  $\tau_{-i}$ . As a consequence, agent  $i$ 's expectation of  $\theta_i$  is

$$E_i(\theta_i) = \frac{\tau_i}{1 + \tau_i}(\theta_i + \eta_i),$$

while the market expectation of  $\theta_i$  is

$$E_{-i}(\theta_i) = \frac{\tau_{-i}}{1 + \tau_{-i}}(\theta_i + \eta_{-i}).$$

Finally, agent  $i$ 's expectation of the market expectation is

$$E_i[E_{-i}(\theta_i)] = \frac{\tau_{-i}}{1 + \tau_{-i}} \frac{\tau_i}{1 + \tau_i}(\theta_i + \eta_i),$$

which implies that  $E_i(E_{-i}(\theta_i))$  is different from the prior (which was assumed equal to zero), if and only if an agent is informed *both* as a consumer-worker and a trader.

Now we can compute the unconditional variance of *the expected*  $k(\theta_i)$ , which captures the consumption volatility that can be reacted upon by choosing labor optimally,

$$\begin{aligned} V(E_i(k(\theta_i))) &= V\left(\left(\beta + (1 - \beta) \frac{\tau_{-i}}{1 + \tau_{-i}}\right) \frac{\tau_i}{1 + \tau_i}(\theta_i + \eta_i)\right) = \\ &= \left(\beta + (1 - \beta) \frac{\tau_{-i}}{1 + \tau_{-i}}\right)^2 \frac{\tau_i}{1 + \tau_i}, \end{aligned} \quad (20)$$

which is increasing in both  $\tau_i$  and  $\tau_{-i}$ . Intuitively, the own precision,  $\tau_i$ , increases the ex-ante volatility of labor allocations that respond to more precise (and then more extreme) signals. The precision about others' productivity,  $\tau_{-i}$  increases the volatility of market expectations, and then the volatility of consumption conditional on a specific labor choice.

We can also note that, *ceteris paribus*, the higher the needs for trading (the lower is  $\beta$ ), the weaker is the reaction of intermediate capital to the realized fundamental  $\theta_i$ . If the agent believes that a large fraction of intermediate capital will be generated with own raw capital the volatility of labor choices will be highly responsive to the fundamental  $\theta_i$ . If not, the volatility of labor will be more responsive to what the

agent expects of the market expectations of  $\theta_i$  and less responsive to what the own agent's expectation about the own productivity.

We compute now the volatility of the individual forecast errors about  $k(\theta_i)$ , which captures the consumption volatility that cannot be reacted upon by choosing labor optimally,

$$\begin{aligned} V_i(k(\theta_i)) &= V\left(\left(\beta + (1 - \beta) \frac{\tau_{-i}}{1 + \tau_{-i}}\right) \left(\theta_i - \frac{\tau_i}{1 + \tau_i} (\theta_i + \eta_i)\right) + (1 - \beta) \frac{\tau_{-i}}{1 + \tau_{-i}} \eta_{-i}\right) = \\ &= \left(\beta + (1 - \beta) \frac{\tau_{-i}}{1 + \tau_{-i}}\right)^2 \frac{1}{1 + \tau_i} + (1 - \beta)^2 \frac{\tau_{-i}}{(1 + \tau_{-i})^2} \end{aligned} \quad (21)$$

which is decreasing in  $\tau_i$  but increasing in  $\tau_{-i}$ . While precision about other agents productivities still increases the volatility of market prices and consumption conditional on a labor choice, the own precision tends to decrease the forecasting errors about market expectations, therefore about market prices.

Now we can resort to the law of total variance to obtain the unconditional variance of  $k(\theta_i)$  as,

$$V(k(\theta_i)) = V(E(k(\theta_i))) + V_i(k(\theta_i)) = \left(\beta + (1 - \beta) \frac{\tau_{-i}}{1 + \tau_{-i}}\right)^2 + (1 - \beta)^2 \frac{\tau_{-i}}{(1 + \tau_{-i})^2}, \quad (22)$$

which is increasing in  $\tau_{-i}$  and  $\beta$ , meaning that smaller fluctuations are associated with high reliance on an uninformed market. Notice that this expression is the generalization of equation (12) under perfect information, in which  $\tau_{-i} = \infty$ .

Another way to decompose the unconditional volatility of  $k(\theta_i)$  is by the sum of the variance that comes from productivity and the variance that comes from noise. We can rewrite the unconditional variance as

$$V(k(\theta_i)) \equiv \underbrace{\left(\beta + (1 - \beta) \frac{\tau_{-i}}{1 + \tau_{-i}}\right)^2}_{V(k(\theta_i)|_\theta)} + \underbrace{(1 - \beta)^2 \frac{\tau_{-i}}{(1 + \tau_{-i})^2}}_{V(k(\theta_i)|_\eta)} = \frac{\beta^2 + \tau_{-i}}{1 + \tau_{-i}}.$$

While  $V(k(\theta_i))|_{\theta}$  measures the volatility due to  $\theta_i$ ,  $V(k(\theta_i))|_{\eta}$  the volatility due to the presence of idiosyncratic noise into the price.

This decomposition is relevant to highlight that the own precision,  $\tau_i$  does not change the unconditional variance of  $k(\theta_i)$ , just the weight of its two components. This result is a version of the externality of own information acquisition onto other agents' labor choices that is channeled through the market, which affects the unconditional variance of intermediate capital. Formally,

**Remark 1.** *The price externality from individual information acquisition is captured by*

$$\frac{\partial V(k(\theta_i))}{\partial \tau_{-i}} = \frac{1 - \beta^2}{(1 + \tau_{-i})^2} > 0,$$

where

$$\begin{aligned} \frac{\partial V(k(\theta_i))|_{\theta}}{\partial \tau_{-i}} &= \frac{2(1 - \beta)(\beta + \tau_{-i})}{(1 + \tau_{-i})^3} > 0 \\ \frac{\partial V(k(\theta_i))|_{\eta}}{\partial \tau_{-i}} &= \frac{(1 - \beta)^2(1 - \tau_{-i})}{(1 + \tau_{-i})^3} > 0 \quad \text{iff } \tau < 1 \end{aligned}$$

In words, the market price becomes more volatile with a more informed market, and in particular, the increase in unconditional volatility of  $k(\theta_i)$  is mainly driven by the comovement of market prices with fundamental productivities.

## 4.1 Optimal Information Precision

Denote  $\tau_i = \tau$  and  $\tau_{-i} = \tau_h$  for all agents and assume the social planner can manage the two precisions  $\tau$  and  $\tau_h$  independently, the planner's problem is

$$\max_{\{\tau, \tau_h\}} \mathbf{V}(\theta_i)$$

The following proposition characterizes the solution.

**Proposition 3.**  $\tau^*(\tau_h) \rightarrow \infty$  for any  $\sigma, \beta$  and  $\tau_h$  and

$$\begin{aligned}\tau_h^*(\tau) &\rightarrow \infty \text{ with } 0 < \sigma < 1 \\ \tau_h^*(\tau) &= 0 \text{ with } \sigma > 1\end{aligned}$$

for any  $\beta < 1$  and  $\tau$ .

*Proof.* Postponed to Appendix A2. ■

The social planner's problem have a “bang-bang” solution. The planner would always like agents to have perfect information about their own raw capital and others; raw capital (this is  $\tau = \infty$  and  $\tau_h = \infty$ ) when  $\sigma < 1$  and perfect information about their own raw capital but no information about others' raw capital (this is  $\tau = \infty$  and  $\tau_h = 0$ ) when  $\sigma > 1$ . Intuitively the planner would like agents to have information to choose labor optimally but would like to avoid them having information that generates volatility in the market after labor has been chosen. This way the planner would take advantage of the positive face of information avoiding the negative face of information.

Based on this benchmark, which highlights where the positive and negative faces of information reside, we turn back to our main setting. First, the planner cannot discriminate between different precision provision about different type of raw capital and can either provide the *information technology* or not, such that giving agents the possibility of accessing and interpreting information about the own raw capital, the planner cannot avoid agents from accessing and interpreting information about others' raw capital. For simplicity, we assume symmetry on signals precision,  $\tau = \tau_h$ . Therefore the planner chooses between providing the free signals to agents ( $\tau_P = \tau$ ) or not ( $\tau_P = 0$ ), such that its problem becomes

$$\max_{\tau_P \in \{0, \tau\}} \mathbf{V}(\theta_i)$$

The next proposition characterizes the solution.

**Proposition 4.** *The planner wants to provide to agents free signals with a precision  $\tau$  about domestic and foreign raw capital if and only if*

$$\begin{aligned} \tau > 0 & \quad \text{for } \sigma < 1 \\ \tau > \frac{\alpha - \beta}{1 - \alpha} & \quad \text{for } \sigma > 1. \end{aligned}$$

where  $\alpha = \sqrt{\frac{\sigma}{\sigma-1}(1 - \beta^2)}$ .

*Proof.* Postponed to Appendix A3. ■

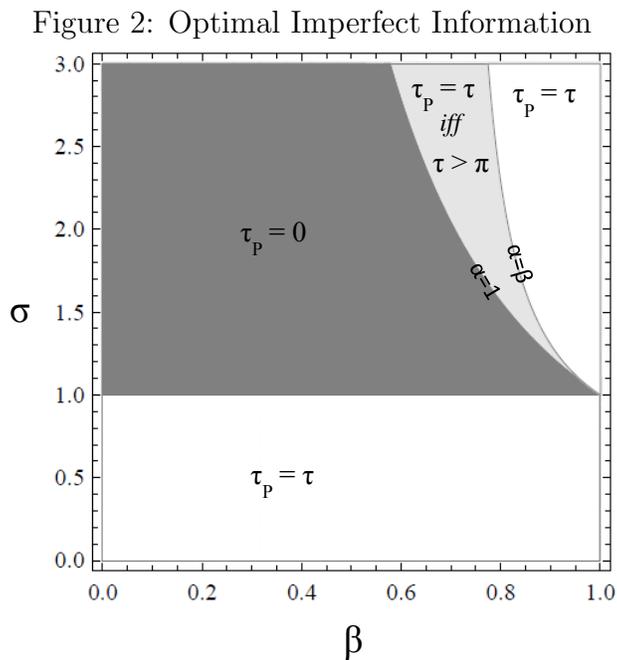
Notice first that, as in our benchmark with perfect information, there is a large difference in the incentives to provide costless information, even though imperfect, if  $\sigma < 1$  and if  $\sigma > 1$ . When  $\sigma < 1$  welfare increases in the volatility of intermediate capital, and regardless of other parameters, the planner would rather provide to agents as much information as possible, even of low precision. In contrast, when  $\sigma > 1$  the unconstrained planner would like to provide to agents signals about the own raw capital but about others' raw capital. Then the constraint that forces the planner to provide either both signals of a given precision or no signal at all introduces a trade-off for the planner when  $\sigma > 1$ .

The trade-off manifests itself as follows. The parameter  $\alpha$  declines with  $\sigma$  given  $\beta$ , while the threshold of information precision increases with  $\alpha$ . Then an increase in  $\sigma$  decreases the information precision at which the planner is willing to handle the information to agents.<sup>4</sup> Similarly, for a given level of risk aversion, when  $\beta$  is large the first effect dominates and the trade-off biases towards acquiring information, as the forecast errors from trading become less relevant. This is why information becomes more likely when  $\beta$  is larger, which can also be easily seen in the proposition. The parameter  $\alpha$  declines with  $\beta$  given  $\sigma$ , while the threshold of information precision increases with  $\alpha$  and decreases with  $\beta$ . Then an increase in  $\beta$  leads to a decline in the information precision required for the planner to provide free signals of a given precision to agents.

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<sup>4</sup>As in the benchmark with perfect information, whenever  $\sigma < \frac{1}{\beta^2}$  the planner would only provide information if  $\tau > \infty$ .

Figure 2 shows the conditions for the acquisition of costless information by the social planner for different  $\beta$  and  $\sigma$  configurations.



As in the analysis with perfect information, it is also the case that, when information is costless (this is,  $c = 0$ ), individuals strictly prefer some information than no information. This outcome is only socially optimal for all  $\sigma$  when  $\beta = 1$ . As long as there are trading needs, there is some range of risk aversion for which the planner would rather not provide information to agents.

## 4.2 Non Monotonicity of Precision on Welfare.

The case of  $\sigma \rightarrow \infty$  allows to uncover an interesting non-monotonicity of the precision of information on welfare. Recall that in such case

$$V_i(k(\theta_i)) = \left( \beta + (1 - \beta) \frac{\tau}{1 + \tau} \right)^2 \frac{1}{1 + \tau} + (1 - \beta)^2 \frac{\tau}{1 + \tau} \frac{1}{1 + \tau}.$$

Both additive terms are composed by two multiplicative factors reflecting the two faces of information: the variance of the expectation ( $\tau/(1 + \tau)$ ) and the variance of the forecast error ( $1/(1 + \tau)$ ). On the one hand, the volatility of the expectation is increasing in information acquisition; on the other hand, the forecast error variance is instead decreasing in information acquisition.

Because of the interaction of these two factors we have the following.

**Proposition 5.** *At the limit of  $\sigma \rightarrow \infty$ , there exists a finite  $\bar{\tau}(\beta)$  such that*

$$\lim_{\sigma \rightarrow \infty} \frac{\partial \mathbf{V}(\theta_i)}{\partial \tau} < 0$$

*for any  $\tau < \bar{\tau}(\beta)$  if and only if  $\beta < \sqrt{1/2}$ ; otherwise*

$$\lim_{\sigma \rightarrow \infty} \frac{\partial \mathbf{V}(\theta_i)}{\partial \tau} \geq 0.$$

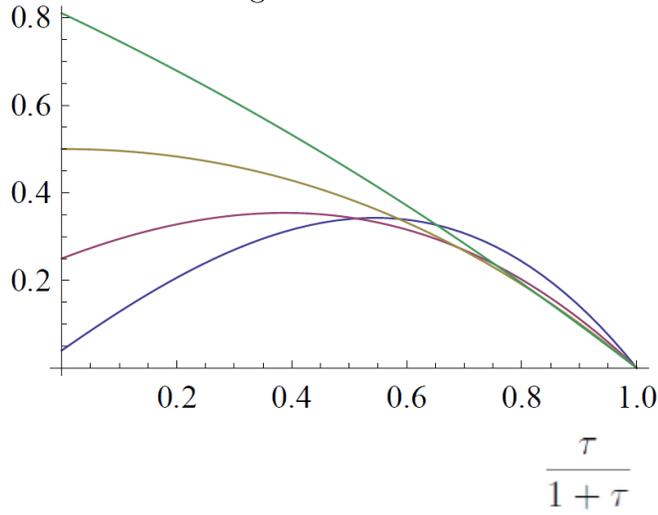
*Proof.* Postponed to Appendix A4.

■

The proposition establishes that the social loss function is not monotonically decreasing in the precision of the signal. This occurs because at low level of precision the volatility of  $k(\theta)$  can increase faster than the ability to forecast it. Figure 3 illustrates this result for different levels of  $\beta$  (to be more precise, 0.9,  $\sqrt{1/2}$ , 0.5 and 0.2). As  $\beta$  decreases, the forecast error variance without any information decreases; however the minimal forecast error variance is obtained with infinite precision with any  $\beta$  value. Note that for  $\beta$  values below  $\sqrt{1/2}$  welfare is non-monotonic in precision, as stated by our proposition. For  $\beta$  values above  $\sqrt{1/2}$  welfare is monotonic on precision and the planner maximizes welfare (minimizes  $\mathbf{V}(\theta_i)$ ) by providing agents with signals of any positive precision instead of not providing them any signal.

Notice that this is consistent with proposition 4. When  $\sigma \rightarrow \infty$ ,  $\alpha \leq \beta$  when  $\beta \geq \sqrt{1/2}$ , which implies that the planner will provide any signal with positive

Figure 3: Forecast error variance



precision to agents. When  $\beta < \sqrt{1/2}$ , then  $\alpha > \beta$  and the planner provides the free signals to the agents as long as those are precise enough.

This is also consistent with the previous analysis of free perfect information. When  $\sigma \rightarrow \infty$  it is always socially desired to have free perfect information as long as  $\beta > 0$ . This is also clear from Figure 3, as  $\tau = \infty$  minimizes the forecast error variance for all levels of  $\beta$  as  $\sigma \rightarrow \infty$ .

## 5 Costly Information

In this section we focus on individual choices when information is costly. In contrast to our previous benchmarks, in which information was free and then agents always want to access it, this is not the case when acquiring information represents a cost for the agents. Even though in principle costly information may discourage individuals to acquire excessive information and realign incentives with those of the planner, we show that costly information introduces the possibility of multiple equilibria, and that not all equilibria are aligned with an efficient outcome, sometimes even implementing excessive opacity.

We assume in this section that information is perfect, but has a cost  $c > 0$  to

acquire in terms of intermediate capital, then  $c_i \in \{0, c\}$ . Expected utility can then be rewritten as

$$E[U_i] = \frac{\sigma}{1-\sigma} e^{\frac{(1-\sigma)^2}{2\sigma} \left( \underbrace{\frac{1}{\sigma} V(E_i(k(\theta_i)))}_{V(k(\theta_i)) - V_i(k(\theta_i))} + V_i(k(\theta_i)) + \frac{2}{1-\sigma}(\kappa - c_i) \right)}, \quad (23)$$

**Planner's Problem.** The planner can still choose whether to provide information to the agents, but now doing so is costly. Based on the previous equation, the welfare criterion is,

$$\mathbf{V}_{c_i}(\theta_i) \equiv \text{sign}(1-\sigma) \left( \frac{1}{\sigma} V(k(\theta_i)) - \frac{1-\sigma}{\sigma} V_i(k(\theta_i)) - \frac{2}{1-\sigma} c_i \right)$$

The next proposition characterizes the condition, in terms of information costs, under which information is efficient.

**Proposition 6.** *Information acquisition is socially desirable when*

$$c \leq \frac{1-\sigma}{2} \left( \frac{1}{\sigma} - \beta^2 \right)$$

*and socially detrimental otherwise.*

*Proof.* As in the proof for Proposition 2, without information  $\mathbf{V}(\theta_i) = \text{sign}(1-\sigma)\beta^2$ . When full information is costly, however, we need to deduct the information cost, so  $\mathbf{V}(\theta_i) = \text{sign}(1-\sigma) \left( \frac{1}{\sigma} - \frac{2}{1-\sigma} c \right)$ . The Proposition follows from comparing these two values of the welfare criteria. ■

The intuition for this result remains identical to that in the benchmark. Indeed, the condition is the same as Proposition 2 when  $c = 0$  and information is socially preferred as long as information is not too costly.

**Individual's Problem and Equilibrium.** We focus on symmetric equilibria. Parallel to how we define a welfare criterion for the planner we can define a utility

criterion for an individual as

$$\mathbf{U}_{c_i|c_{-i}}(\theta_i) \equiv \text{sign}(1 - \sigma) \left( \frac{1}{\sigma} V(k(\theta_i)) - \frac{1 - \sigma}{\sigma} V_i(k(\theta_i)) - \frac{2}{1 - \sigma} \left( c_i - \frac{\Delta_{i|c_{-i}}}{\gamma} \right) \right).$$

This expression is obtained just from plugging  $\kappa_i = \kappa + \frac{\Delta_{i|c_{-i}}}{\gamma} - c_i$ , in the expected utility, where  $\Delta_{i|c_{-i}}$  is the individual expected trading gains that could come from information acquisition, *conditional* on whether other agents have acquired information ( $c_{-i} = c$ ) or not ( $c_{-i} = 0$ ). This extra term that comes from potential trading gains was naturally absent in the planner's choice. The following proposition characterizes the conditions under which all agents are informed in equilibrium (*informed equilibrium*) or none are (*uninformed equilibrium*).

**Proposition 7.** *A symmetric equilibrium with information acquisition exists if*

$$c \leq \frac{(1 - \sigma)^2}{2\sigma} + \frac{\Delta_{i|c}}{\gamma}.$$

*A symmetric equilibrium without information acquisition exists if*

$$c \geq \beta^2 \frac{(1 - \sigma)^2}{2\sigma} + \frac{\Delta_{i|0}}{\gamma}.$$

*Proof.* Assume first all agents are informed (this is,  $c_{-i} = c$ ). If agent  $i$  acquires information, the utility criterion is  $\mathbf{U}_{c|c}(\theta_i) = \text{sign}(1 - \sigma) \left[ \frac{1}{\sigma} - \frac{2}{1 - \sigma} c \right]$ . If agent  $i$  does not acquire information, the utility criterion is  $\mathbf{U}_{0|c}(\theta_i) = \text{sign}(1 - \sigma) \left[ 1 - \frac{2}{1 - \sigma} \frac{\Delta_{i|c}}{\gamma} \right]$ . The condition in the proposition comes from imposing that

$$\mathbf{U}_{c|c}(\theta_i) \geq \mathbf{U}_{0|c}(\theta_i).$$

Assume now that no agent is informed (this is,  $c_{-i} = 0$ ). If agent  $i$  acquires information, the utility criterion is  $\mathbf{U}_{c|0}(\theta_i) = \text{sign}(1 - \sigma) \left[ \frac{\beta^2}{\sigma} - \frac{2}{1 - \sigma} \left( c - \frac{\Delta_{i|0}}{\gamma} \right) \right]$ . If agent  $i$  does not acquire information, the utility criterion is  $\mathbf{U}_{0|0}(\theta_i) = \text{sign}(1 - \sigma) \beta^2$ . The

condition in the proposition comes from imposing that

$$U_{c|0}(\theta_i) \leq U_{0|0}(\theta_i).$$

■

Intuitively, an *informed equilibrium* exists when the benefit of not acquiring information (the cost  $c$ ) is not greater than the cost of not acquiring information given that all other agents do. This cost is given by the expected trading losses from selling and buying from other, informed, agents (captured by  $\frac{\Delta_{i|c}}{\gamma}$ ) and by being unable to react to the realized productivity of raw capital, *both used and traded*.

Similarly, an *uninformed equilibrium* exists when the cost of acquiring information (the cost  $c$ ) is not smaller than the benefit of acquiring information given that no other agent does. This benefit is given by the expected trading gains from selling and buying from other, uninformed, agents (captured by  $\frac{\Delta_{i|c}}{\gamma}$ ) and by being able to react to the realized productivity of raw capital, *but only the used* (captured by the term  $\beta^2$ ), as the traded raw capital is can be sold at the known expected productivity given than no other agent acquires information.

The possibility of multiplicity comes from the asymmetry on the benefits of information as a function of the information in the hands of other agents. Take the case, for instance, in which there are no learning from prices in the market for raw capital, and then informational trading gains are independent on the fraction of informed agents in the market (this is,  $\Delta_{i|c} = \Delta_{i|0} = \frac{1}{2}$ ). This is the cleanest case to discuss the source of multiplicity because the incentives to acquire information for trading gains are symmetric when all or no agent has information.

The asymmetry on the benefits of information comes on the variance that agents can react to by acquiring information. The costs of not having information when all others have information are larger than the benefits of acquiring information when no other agent has information. When all agents have information, the agent faces the variance of own raw capital productivity regardless of whether it uses it or sells it, which determines the gains from being able to adjust labor. When no agent has information, the agent can also react to the variance of own raw capital productivity,

but only of the fraction that it uses, not the one that he sells which is determined by other uninformed agents. The largest is the fraction of own raw capital that the agent sells (the lower is  $\beta$ ) the lower the incentives to acquire information when no other agent does.

Learning from prices introduces a force for non-existence, as highlighted by Grossman and Stiglitz (1980). When all agents are informed prices are perfectly revealing and there are no losses from being the single uninformed agent ( $\Delta_{i|c} = 0$ ). In contrast, when no other agent is informed there are trading gains from having information and taking advantage of other traders ( $\Delta_{i|c} = \frac{1}{2}$ ). This force introduces an asymmetry on the opposite direction, incentivizing agents to acquire information more when no other agent is informed than when all agents are informed.

This discussion highlights that our mechanism not only introduces the possibility of multiplicity but also a force that justifies information in a setting in which information is costly and there are no noise traders, so prices are perfectly revealing.

Figure 4 shows the range of equilibria when  $c = 0.1$  (using the same parameters as in previous figures). In constructing this figure we have assumed no learning from prices so we can focus on the role of asymmetric incentives to acquire information that come only from our mechanism. The white regions show the parameter combination under which only the informed equilibrium ( $I$ ) exists, the dark grey the combination under which only the uninformed equilibrium ( $U$ ) exists, and the light grey the combination under which both exist.

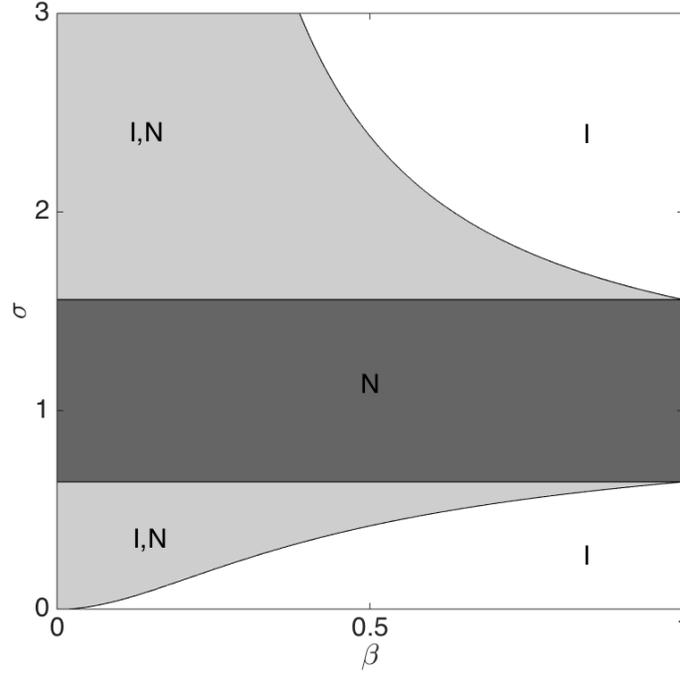
The next two corollaries of the previous propositions summarize the efficiency implications of this multiplicity. In contrast to the case of free information, in which the only source of inefficiency comes from excessive information, here excessive opacity can also be sustained in equilibrium.

**Corollary 2.** *There is room for inefficient information in the economy when*

$$\frac{\Delta_{i|c}}{\gamma} > \frac{(1 - \sigma)(1 - \beta^2)}{2}$$

*Proof.* This condition is obtained from comparing the conditions under which information is inefficient and those that sustain information in equilibrium. Information

Figure 4: Optimal Imperfect Information



is inefficient when

$$c > \frac{(1 - \sigma)^2}{2\sigma} + \frac{(1 - \sigma)(1 - \beta^2)}{2},$$

from rewriting the condition in proposition 6. The informed equilibrium exists when

$$c \leq \frac{(1 - \sigma)^2}{2\sigma} + \frac{\Delta_{i|c}}{\gamma},$$

from proposition 7. Both conditions are mutually consistent if the condition in the corollary holds. ■

In the case of price learning, for instance,  $\Delta_{i|c} = 0$  and there is no room for excessive information when  $\sigma < 1$  but there is always room for this inefficiency when  $\sigma > 1$ , consistent with our main costless information benchmark.

**Corollary 3.** *There is room for inefficient opacity in the economy when*

$$\frac{\Delta_{i|0}}{\gamma} < \frac{(1-\sigma)(1-\beta^2)}{2\sigma}$$

*Proof.* This condition is obtained from comparing the conditions under which information is efficient and those that prevent information in equilibrium. Information is efficient when

$$c \leq \beta^2 \frac{(1-\sigma)^2}{2\sigma} + \frac{(1-\sigma)(1-\beta^2)}{2\sigma},$$

from rewriting the condition in proposition 6. The uninformed equilibrium exists as long as

$$c > \beta^2 \frac{(1-\sigma)^2}{2\sigma} + \frac{\Delta_{i|0}}{\gamma},$$

from proposition 7. Both conditions are mutually consistent if the condition in the corollary holds. ■

Also in the case of learning from prices,  $\Delta_{i|0} = \frac{1}{2}$  and there is no room for excessive opacity if  $\sigma > 1$ . There is room for it, however, when  $\sigma < 1$  only if  $\frac{\sigma}{1-\sigma} < \gamma(1-\beta^2)$ . The logic from this result is that the planner values more information as  $\sigma$  and  $\beta$  are smaller but individuals value it less as  $\gamma$  is large, which depletes trading gains. This never happens when  $c = 0$  as individuals always prefer information when costless.

## 6 Conclusions

Information has two faces. On the one hand it improves the allocation of resources because knowing the productivity of inputs allows to better choose their combination. On the other hand it generates volatility in the trading prices of those resources after allocation has been decided. The positive face of information is the increase in insurance provided by the allocation of resources. The negative face is the decline in insurance provided by market diversification through trading.

While the positive side of information is internalized by individuals, the negative side is not. This negative externality generates both multiple equilibria and ineffi-

ciencies. In particular some equilibria display excessive information acquisition. This inferior equilibria with too much information is more likely to happen when information is cheap, individuals are not extremely risk averse and the needs for trading are relatively large. In this scenario a planner would rather give up on optimal investments than facing a large volatility in consumption generated by forecast errors in trading. Some equilibria also display too little information. This is in general the case when risk aversion is less than one and the trading needs are large enough.

A more counterintuitive result is that even free and perfect information may not be desirable from a social perspective. When the insurance from reallocation (captured by risk aversion) is low when compared to the insurance from market diversification (captured by trading needs), then a planner would rather abscond information even when that is perfectly precise and absolutely costless. When the gains from reallocation are low, the planner would prefer to lose in productivity gains but to gain in reducing in consumption volatility. Even more surprising is that the combination of parameters that are empirically plausible suggests that information, even free and perfect, is socially undesirable as its increase in market volatility does not compensate the improvement in the allocation of resources.

These results arise from imposing a plausible information structure in which an agent cannot avoid acquiring signals (or making inferences) about inputs they may buy when acquiring signals about their own inputs. We capture this structure by referring to an information technology. If the planner could freely choose an information structure it would choose one that only provides signals about the own goods or that impedes inferences about other agents' goods.

## References

- Allen, Franklin, Stephen Morris, and Hyun Song Shin. 2006. "Beauty Contests and Iterated Expectations in Asset Markets." *Review of Financial Studies* 19:719–752.
- Angeletos, George-Marios, and Alessandro Pavan. 2007. "Efficient Use of Informa-

- tion and Social Value of Information.” *Econometrica* 75:1103–1142.
- Barlevy, Gadi, and Pietro Veronesi. 2000. “Information Acquisition in Financial Markets.” *Review of Economic Studies* 67:79–90.
- Basu, Susanto. 1995. “Intermediate Goods and Business Cycles: Implications for Productivity and Welfare.” *American Economic Review* 85:512–531.
- Bester, Helmut. 1985. “Screening vs. Rationing in Credit Markets with Imperfect Information.” *American Economic Review* 71:393–410.
- Campbel, Tim, and William Kracaw. 1980. “Information Production, Market Signaling and the Theory of Financial Intermediation.” *Journal of Finance* 35:863–881.
- Caramp, Nicolas. 2017. “Sowing the Seeds of Financial Crises: Endogenous Asset Creation and Adverse Selection.” Working Paper, MIT.
- Ciccone, Antonio. 2002. “Input chains and industrialization.” *Review of Economic Studies* 69:565–587.
- Colombo, Luca, Gianluca Femminis, and Alessandro Pavan. 2014. “Information Acquisition and Welfare.” *Review of Economic Studies* 81:1438–1483.
- Dang, Tri Vi, Gary Gorton, Bengt Holmström, and Guillermo Ordoñez. 2014. “Banks as Secret Keepers.” *American Economic Review*, Forthcoming.
- Diamond, Douglas. 1984. “Financial Intermediation and Delegated Monitoring.” *Review of Economic Studies* 51:393–414.
- . 1991. “Monitoring and Reputation: The Choice between Bank Loans and Directly Placed Debt.” *Journal of Political Economy* 99:689–721.
- Dow, James, Itay Goldstein, and Alexander Guembel. 2017. “Incentives for Information Production in Markets where Prices Affect Real Investment.” *Journal of the European Economic Association*, forthcoming.
- Gaballo, Gaetano. 2016. “Rational Inattention to News: The Perils of Forward Guidance.” *American Economic Journal: Macroeconomics* 8:42–97.

- Goldstein, Itay, and Liyan Yang. 2015. "Information Diversity and Complementarities in Trading and Information Acquisition." *Journal of Finance* 70:1723–1765.
- Gorton, Gary, and George Pennacchi. 1990. "Financial Intermediaries and Liquidity Creation." *Journal of Finance* 45 (1): 49–72.
- Grossman, Sanford, and Joseph Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70:393–408.
- Hellwig, Christian, and Laura Veldkamp. 2009. "Knowing What Others Know: Coordination Motives in Information Acquisition." *Review of Economic Studies* 76:223–251.
- Hirshleifer, Jack. 1971. "The Private and Social Value of Information and the Reward to Inventive Activity." *American Economic Review* 61 (4): 561–674.
- Jones, Charles. 2010. "Intermediate Goods and Weak Links in the Theory of Economic Development." Working Paper, Stanford University.
- Kurlat, Pablo, and Laura Veldkamp. 2015. "Should We Regulate Financial Information?" *Journal of Economic Theory* 158:697–720.
- Leland, Hayne, and David Pyle. 1977. "Informational Asymmetries, Financial Structure, and Financial Intermediation." *Journal of Finance* 31:371–387.
- Morris, Stephen, and Hyun Song Shin. 2002. "Social Value of Public Information." *American Economic Review* 92 (5): 1521–1534.
- Vanasco, Victoria. 2017. "The Downside of Asset Screening for Market Liquidity." *Journal of Finance*, forthcoming.
- Vives, Xavier. 2014. "On The Possibility Of Informationally Efficient Markets." *Journal of the European Economic Association* 12:1200–1239.

# Appendix

## A1 Derivation of the welfare function

Here we derive our welfare function assuming that  $k(\theta) = k(\theta) + \kappa$  with  $k(\theta) \sim N(0, 1)$  and  $\kappa > 0$  being a deterministic component. Given the properties of the log-normal distributions we also have

$$\begin{aligned}
 U_i &= \frac{e^{(1-\sigma)(k(\theta)+\kappa)} E_i[e^{(1-\sigma)(k(\theta)+\kappa)}]^{1-\sigma}}{1-\sigma} - E_i[e^{(1-\sigma)(k(\theta)+\kappa)}]^{1-\sigma} = \\
 &= \frac{e^{(1-\sigma)(k(\theta)+\kappa)} e^{\frac{(1-\sigma)^2}{\sigma}(E_i(k(\theta))+\kappa)+\frac{(1-\sigma)^3}{2\sigma}V_i(k(\theta))}}{1-\sigma} \\
 &\quad - e^{\frac{(1-\sigma)}{\sigma}(E_i(k(\theta))+\kappa)+\frac{(1-\sigma)^2}{2\sigma}V_i(k(\theta))}
 \end{aligned}$$

and then take the unconditional expectation, which gives

$$\begin{aligned}
 E[U_i] &= E \left[ \frac{e^{(1-\sigma)(k(\theta)+\kappa)} e^{\frac{(1-\sigma)^2}{\sigma}(E_i(k(\theta))+\kappa)+\frac{(1-\sigma)^3}{2\sigma}V_i(k(\theta))}}{1-\sigma} \right] \\
 &\quad - E \left[ e^{\frac{(1-\sigma)}{\sigma}(E_i(k(\theta))+\kappa)+\frac{(1-\sigma)^2}{2\sigma}V_i(k(\theta))} \right] \\
 &= \frac{e^{\frac{1}{2}V_0 \left( (1-\sigma)k(\theta) + \frac{(1-\sigma)^2}{\sigma}E_i(k(\theta)) \right) + \frac{(1-\sigma)^3}{2\sigma}V_i(k(\theta)) + \frac{1-\sigma}{\sigma}\kappa}}{1-\sigma} \\
 &\quad - e^{\frac{(1-\sigma)^2}{2\sigma^2}V_0(E_i(k(\theta))) + \frac{(1-\sigma)^2}{2\sigma}V_i(k(\theta)) + \frac{1-\sigma}{\sigma}\kappa} \\
 &= \frac{e^{\frac{1}{2}V_0 \left( (1-\sigma)(k(\theta) - E_i(k(\theta))) + \frac{1-\sigma}{\sigma}E_i(k(\theta)) \right) + \frac{(1-\sigma)^3}{2\sigma}V_i(k(\theta)) + \frac{1-\sigma}{\sigma}\kappa}}{1-\sigma} \\
 &\quad - e^{\frac{(1-\sigma)^2}{2\sigma^2}V_0(E_i(k(\theta))) + \frac{(1-\sigma)^2}{2\sigma}V_i(k(\theta)) + \frac{1-\sigma}{\sigma}\kappa}
 \end{aligned}$$

Exploiting the fact that  $E[E(x|y)(x - E(x|y))] = 0$  and that  $E[(x - E(x|y))^2] = V(x|y)$  we further have

$$\begin{aligned}
E[U_i] &= \frac{e^{\frac{(1-\sigma)^2}{2} V_i(k(\theta)) + \frac{(1-\sigma)^2}{2\sigma^2} V_0(E_i(k(\theta))) + \frac{(1-\sigma)^3}{2\sigma} V_i(k(\theta)) + \frac{1-\sigma}{\sigma} \kappa}}{1 - \sigma} \\
&\quad - \frac{e^{\frac{(1-\sigma)^2}{2\sigma^2} V_0(E_i(k(\theta))) + \frac{(1-\sigma)^2}{2\sigma} V_i(k(\theta)) + \frac{1-\sigma}{\sigma} \kappa}}{1 - \sigma} \\
&= \frac{e^{\frac{(1-\sigma)^2}{2\sigma^2} V_0(E_i(k(\theta))) + \frac{(1-\sigma)^2}{2\sigma} V_i(k(\theta)) + \frac{1-\sigma}{\sigma} \kappa}}{1 - \sigma} \\
&\quad - \frac{e^{\frac{(1-\sigma)^2}{2\sigma^2} V_0(E_i(k(\theta))) + \frac{(1-\sigma)^2}{2\sigma} V_i(k(\theta)) + \frac{1-\sigma}{\sigma} \kappa}}{1 - \sigma} \\
&= \frac{\sigma}{1 - \sigma} e^{\frac{(1-\sigma)^2}{2\sigma^2} V_0(E_i(k(\theta))) + \frac{(1-\sigma)^2}{2\sigma} V_i(k(\theta)) + \frac{1-\sigma}{\sigma} \kappa} = \\
&= \frac{\sigma}{1 - \sigma} e^{\frac{(1-\sigma)^2}{2\sigma} \left( \frac{1}{\sigma} V_0(E_i(k(\theta))) + V_i(k(\theta)) + \frac{2}{1-\sigma} \kappa \right)}.
\end{aligned}$$

## A2 Proof Proposition 3

Let us calculate first the impact of private precision on welfare

$$\frac{\partial \left( \left( \beta + (1 - \beta) \frac{\tau_h}{1 + \tau_h} \right)^2 \left( \frac{1}{\sigma} \frac{\tau}{1 + \tau} + \frac{1}{1 + \tau} \right) \right)}{\partial \tau} = \frac{1 - \sigma}{\sigma} \frac{(\beta + \tau_h)^2}{(\tau + 1)^2 (\tau_h + 1)^2}.$$

This is positive with  $\sigma < 1$  and negative otherwise, so the conclusion easily obtains.

Let us come now to the impact of the foreign precision

$$\begin{aligned}
\frac{\partial \left( \left( \beta + (1 - \beta) \frac{\tau_h}{1 + \tau_h} \right)^2 \left( \frac{1}{\sigma} \frac{\tau}{1 + \tau} + \frac{1}{1 + \tau} \right) \right)}{\partial \tau_h} &= \frac{2(1 - \beta)(\sigma + \tau)(\beta + \tau_h)}{\sigma(\tau + 1)(\tau_h + 1)^3} > 0 \\
\frac{\partial \left( (1 - \beta)^2 \frac{\tau_h}{(1 + \tau_h)^2} \right)}{\partial \tau_h} &= \frac{(1 - \beta)^2 (1 - \tau_h)}{(1 + \tau_h)^3}
\end{aligned}$$

whose sum is positive for any  $\tau_h > \hat{\tau}_h$  where

$$\hat{\tau}_h = -\frac{\sigma(1 + \beta) + (2\beta + (1 - \beta)\sigma)\tau}{\sigma(1 + \beta) + (2 - (1 - \beta)\sigma)\tau}.$$

With  $\hat{\tau}_h < 0$  the impact is always positive; this implies  $\tau_h^* \rightarrow \infty$  with  $\sigma < 1$  and  $\tau_h^* = 0$  with  $\sigma > 1$ .

With  $\hat{\tau}_h > 0$  is initially positive and then negative as  $\tau_h$  increases. In such a case  $\tau_h^* = \hat{\tau}_h$  with  $\sigma < 1$  and either  $\tau_h^* = 0$  or  $\tau_h^* \rightarrow 0$  with  $\sigma > 1$ .

We have  $\hat{\tau}_h < 0$  when

$$\tau < \frac{\sigma(1+\beta)}{\sigma(1-\beta)-2} \quad \text{and} \quad \sigma > \frac{2}{1-\beta}$$

and always with  $\sigma < 2/(1-\beta)$ .

We have  $\hat{\tau}_h > 0$  if and only if

$$\tau > \frac{\sigma(1+\beta)}{\sigma(1-\beta)-2} \quad \text{and} \quad \sigma > \frac{2}{1-\beta}.$$

This gives

$$\begin{aligned} \tau_h^*(\tau) &\rightarrow \infty \quad \text{with} \quad \sigma < 1 \\ \tau_h^*(\tau) &= 0 \quad \text{with} \quad 1 > \sigma > \frac{2}{1-\beta} \\ \tau_h^*(\tau) &= 0 \quad \text{with} \quad \tau < \frac{\sigma(1+\beta)}{\sigma(1-\beta)-2} \quad \text{and} \quad \sigma > \frac{2}{1-\beta} \\ \tau_h^*(\tau) &= 0 \quad \text{with} \quad \tau > \frac{\sigma(1+\beta)}{\sigma(1-\beta)-2} \quad \text{and} \quad \sigma > \frac{2}{1-\beta} \end{aligned}$$

where the last implication obtains because

$$\left( \beta^2 \left( \frac{1}{\sigma} \frac{\tau}{1+\tau} + \frac{1}{1+\tau} \right) \right) < \left( \left( \frac{1}{\sigma} \frac{\tau}{1+\tau} + \frac{1}{1+\tau} \right) + \frac{1}{4}(1-\beta)^2 \right)$$

that is social welfare is higher with  $\tau_h^* = 0$  than with  $\tau_h^* \rightarrow \infty$  when  $\sigma > 1$  (this relation is reversed with  $\sigma < 1$  because  $\text{sign}(1-\sigma)$  flips).

Take now  $\tau^* \rightarrow \infty$  therefore the thesis.

Suppose now  $V_0(k(\theta_i))|_\eta = 0$ . Our conclusions are not affected.

### A3 Proof Proposition 4

The planner's welfare is given by equation (?). Impose  $\tau_{i,j} = \tau_{-i} = \tau$  and  $c_i = 0$ . First we prove the shape of welfare as a function of  $\tau$ . Then we characterize what is the choice of  $\tau_P$  as a function of  $\sigma$  and  $\beta$ .

### Properties of welfare as a function of $\tau$

Compute

$$\frac{\partial \left( (\beta + (1 - \beta) \frac{\tau}{1+\tau})^2 \left( \frac{1}{\sigma} \frac{\tau}{1+\tau} + \frac{1}{1+\tau} \right) \right)}{\partial \tau} = \frac{(\beta + \tau) (2\sigma + \beta + 3\tau - 3\sigma\beta - \sigma\tau - 2\beta\tau)}{\sigma (1 + \tau)^4}$$

and

$$\frac{\partial \left( (1 - \beta)^2 \frac{\tau}{(1+\tau)^2} \right)}{\partial \tau} = \frac{(1 - \beta)^2 (1 - \tau)}{(1 + \tau)^3}$$

so that the overall derivative is given by

$$\frac{\partial \mathbf{V}(\theta_i)}{\partial \tau} = \text{sign}(1 - \sigma) \frac{\varphi(\tau)}{\sigma (\tau + 1)^4}$$

with

$$\varphi(\tau) = (3 - 2\beta - (1 + (1 - \beta)^2) \sigma) \tau^2 + (2\sigma + 4\beta - 2\beta^2 - 4\sigma\beta) \tau + (\sigma - 2\sigma\beta^2 + \beta^2)$$

where notice the numerator is a second-order polynomial in  $\tau$  that we conveniently denote by  $\varphi(\tau)$ . Let us now go through the different cases.

The first case is  $\sigma < 1$ . In this case  $\varphi(\tau)$  is a parable which is open upwards since

$$3 - 2\beta - (1 + (1 - \beta)^2) \sigma > 0 \tag{A1}$$

i.e.

$$\sigma < 1 < \frac{3 - 2\beta}{1 + (1 - \beta)^2}.$$

Moreover the real roots of  $\varphi(\tau)$ , whenever they exist (which occurs with  $\sigma < \beta^2 / (3 - 2\beta^2)$ ) are both negative. To see this notice that, whenever the parable is upwards, for both to have the same sign it must be

$$\sigma + \beta^2 - 2\sigma\beta^2 > 0 \tag{A2}$$

which occurs

$$\text{with } \beta > \sqrt{1/2} \text{ and } \sigma < 1 < \frac{\beta^2}{2\beta^2 - 1}$$

or with  $\beta < \sqrt{1/2}$ ; and that, whenever the parable is upwards, for their sum is negative it must be

$$(2\sigma + 4\beta - 2\beta^2 - 4\sigma\beta) > 0 \tag{A3}$$

which occurs if

$$\text{with } \beta > 1/2 \text{ and } \sigma < 1 < \frac{2(2-\beta)\beta}{2(2\beta-1)}$$

or with  $\beta < 1/2$ . We then conclude that  $\partial \mathbf{V}(\theta_i)/\partial \tau$  must be always positive and strictly increasing in the first quadrant. This implies  $\tau^* \rightarrow \infty$  with  $\sigma < 1$ .

Before proceeding with the second case it is useful to note that

$$1 < \frac{3-2\beta}{1+(1-\beta)^2} < \frac{1}{\beta^2}$$

holds for whatever  $\beta$  and

$$\frac{1}{\beta^2} < \frac{\beta^2}{2\beta^2-1}$$

for  $\beta > \sqrt{1/2}$ .

In the second case is  $\sigma > 1$  two real roots always exists. With

$$1 < \sigma < \frac{3-2\beta}{1+(1-\beta)^2}$$

the parable is still open upwards and roots are both negative. However this implies now  $\tau^* = 0$  as the sign  $(1-\sigma)$  is reverted. With

$$\frac{3-2\beta}{1+(1-\beta)^2} < \sigma < \frac{\beta^2}{|2\beta^2-1|}$$

for whatever  $\beta$ , the parable is now open downwards, which also flips the implications of (A2) and (A3). That is now the roots have a different sign, i.e. one is positive and the other is negative. This means that  $\varphi(\tau)$  is first positive and then becomes negative after a certain finite positive value  $\bar{\tau}$ . This in turn implies that welfare initially decreases and then increases as precision increases. Therefore the solution is  $\tau^* = 0$  when

$$\lim_{\tau \rightarrow 0} \mathbf{V}(\theta_i) = \beta^2 < \lim_{\tau \rightarrow \infty} \mathbf{V}(\theta_i) = \frac{1}{\sigma}$$

and  $\tau^* \rightarrow \infty$  otherwise. Finally when  $\beta > \sqrt{1/2}$  and

$$\frac{\beta^2}{2\beta^2-1} < \sigma$$

the two roots have the same sign. However, for that range we also have that the sum of the roots is negative, so  $\partial \mathbf{V}(\theta_i)/\partial \tau$  must be always negative and strictly decreasing in the first quadrant. This also implies  $\tau^* \rightarrow \infty$ .

### Optimal choice of $\tau_P$

Define  $x = \frac{\tau}{1+\tau}$ . When the planner chooses to avoid agents to have the free signals (then choosing  $\tau_P = 0$ ), welfare is  $\text{sign}(1 - \sigma)\beta^2$ .

When the planner chooses to provide agents the free signals (then choosing  $\tau_P = \tau$ ), welfare is

$$\text{sign}(1 - \sigma) \left[ (\beta + (1 - \beta)x)^2 \left( \frac{x}{\sigma} + (1 - x) \right) + (1 - \beta)^2 x(1 - x) \right]$$

When  $\sigma < 1$  we have shown that welfare is monotonically increasing in  $x$ . Imposing the maximum precision of information,  $x = 1$ , welfare is  $\frac{1}{\sigma} > \beta^2$  always. This is why the planner chooses to provide free signals to the agents as soon as  $\tau > 0$ .

When  $\sigma > 1$  we have shown that welfare is either monotonically decreasing (relative low  $\beta$ ), increasing (relative high  $\beta$ ) or quadratic in  $x$  (for intermediate  $\beta$  levels, first decreasing in  $x$  and then increasing in  $x$ ). Then the planner chooses to give free signals of precision  $\tau$  to agents when

$$\begin{aligned} -\beta^2 &< - \left[ (\beta + (1 - \beta)x)^2 \left( \frac{x}{\sigma} + (1 - x) \right) + (1 - \beta)^2 x(1 - x) \right] \\ \beta^2 \left( \frac{\sigma - 1}{\sigma} \right) &< (1 - \beta)^2 \left[ 1 - x^2 \left( \frac{\sigma - 1}{\sigma} \right) \right] + 2\beta(1 - \beta) \left[ 1 - x \left( \frac{\sigma - 1}{\sigma} \right) \right] \end{aligned}$$

This leads to a quadratic equation

$$(1 - \beta)^2 x^2 + 2\beta(1 - \beta)x + \left[ \beta^2 - \frac{\sigma}{\sigma - 1}(1 - \beta^2) \right] < 0$$

and then, based on the quadratic property of welfare, we have to use the largest root of this quadratic equation, which leads to the condition

$$x > \frac{\alpha - \beta}{1 - \beta}$$

where  $\alpha = \sqrt{\frac{\sigma}{\sigma - 1}(1 - \beta^2)}$ . The condition of  $\tau$  is then

$$\tau > \frac{\alpha - \beta}{1 - \alpha}$$

## A4 Proof Proposition 5

We have

$$\begin{aligned} \lim_{\sigma \rightarrow \infty} \frac{\partial \mathbf{V}(\theta_i)}{\partial \tau} &= - \frac{\partial \left( (\beta + (1 - \beta) \frac{\tau}{1 + \tau})^2 \frac{1}{1 + \tau} + (1 - \beta)^2 \frac{\tau}{1 + \tau} \frac{1}{1 + \tau} \right)}{\partial \tau} = \\ &= \frac{(1 + (1 - \beta)^2) \tau^2 + 2(2\beta - 1) \tau + 2\beta^2 - 1}{(1 + \tau)^4} < 0 \end{aligned}$$

for any  $\tau < \bar{\tau}(\beta)$  with  $\tau(\beta)$  finite, whenever there are two roots with different sign, that is when  $2\beta^2 - 1 < 0$  which requires  $\beta < \sqrt{1/2}$ . This is the only condition as when instead the two roots have the same sign their sum is also negative, i.e.  $(2\beta - 1) > 0$  whenever  $\beta > \sqrt{1/2}$ . Also notice two real roots always exist since the determinant  $3 - 2\beta^2$  is positive for any value of  $\beta < 1$ .

## A5 Partially correlated signals of foreign capital

Here we want to show a version of our model where agents receive partly correlated signals about productivity, but as they learn from financial prices they end up having homogeneous information sets. Hence, this serves as a microfundation for our assumption on perfectly correlated signals about foreign islands. We avoid the well-known Grossman-Stiglitz paradox created by learning from prices by adopting a mechanism inspired by the work of Vives (2014).

Informed investors receive correlated private signals about  $\theta_h$  of the form  $s_{i,j} = \theta_h + \eta_{-h} + \zeta_{i,j}$  where  $\zeta_{i,j} \sim N(0, \tau_\zeta^{-1/2})$  is an i.i.d. shock. For the purpose of this section we follow Vives (2014)' assumption that the productivity of capital  $i$  in the hands of investor  $(h, j)$  is given by  $\theta_i + \zeta_{h,j}$ . This problem is equivalent to the one in the main text when  $\tau_\zeta$  is close to zero. Relaxing this assumption only adds an innocuous constant to investors' profit that depends on  $\tau_\zeta$  but not on  $\tau_h$ .

Given the market structure in our model (in particular no uncertainty of total supply), the capital price  $R_h$  is informative of the average expectation of raw capital  $h$  across investors. Let us suppose that an equivalent price signal denoted by  $\tilde{R}_h$  takes the form

$$\tilde{R}_h = \int E_{h,j}[\theta_h + \zeta_{i,j}] di = \frac{\tau_h}{1 + \tau_h} (\theta_h + \eta_{-h}),$$

which is the same as in our original setting. If this is the case then investors must

form their expectation according to

$$E_{h,j}[\theta_i + \zeta_{i,j}] = \tilde{R}_h + \underbrace{s_{i,j} - \frac{1 + \tau_h}{\tau_h} \tilde{R}_h}_{=\zeta_{i,j}}$$

where notice that  $\zeta_{i,j}$  is perfectly known. So it must be

$$\int E_{h,j}[\theta_h + \zeta_{i,j}] di = \tilde{R}_h$$

Intuitively, even in the perfect information limit we do not have the Grossman-Stiglitz paradox as investors always need to use their own private information to forecast the idiosyncratic component  $\zeta_{i,j}$ .