Implicit Fiscal Guarantee for Monetary Stability*

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PRELIMINARY AND INCOMPLETE

Abstract

This paper looks at the conditions under which an authority with fiscal power can credibly sustain the real value of fiat money. We study a classical OLG monetary model where agents can save in money or in a storage technology with an inefficiently low real return. We introduce an authority that maximizes current agents' welfare and its own consumption by imposing taxes and carrying out open market operations in the money market. We show that, even if the authority can fix a positive term of trade between money and the consumption good at a certain point in time, private money demand can be zero. This is the case when the authority cannot credibly resist the temptation to raise seignorage the period after, once private agents hold money. In particular, when the authority has a flexible tax instrument to secure its own consumption, it will always conduct open market operations to sustain the value of money – no matter the level of benevolence of its objective – leading the monetary equilibrium to be unique. On the contrary, if the authority lacks such a fiscal instrument, the monetary equilibrium realizes only insofar either the authority is sufficiently benevolent or it is already sufficiently endowed with real resources. Our analysis points out to the need of microfoundations to the authority's behavior in the debate on the price level determination and offers a new perspective on the importance of fiscal backing.

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1 Introduction

Recent progress on cryptocurrency payment systems brought into prominence a never settle debate on what makes a fundamentally worthless asset become money and whether a truly private monetary system is possible. Common wisdom has it that monetary stability rests on the credibility of a benevolent authority that guarantees the real value of paper money. This implicit guarantee visualizes in the signature of the governor of the Central Bank - for the Euro - or the Treasury - for the Dollar - and it is commonly believed by a huge number of people in their everyday transactions. But how can these authorities comply with their promises?

In an influential paper Obstfeld and Rogoff (1983) argue that in fact monetary stability requires a minimal involvement by an authority. By guaranteeing an arbitrarily small and probabilistic real redemption value for money the authority makes speculative hyperinflations, along which money progressively looses value, inconsistent with rational behavior of private agents. Thus, in theory ruling out monetary crises is surprisingly simple and cheap! So simple and cheap that it becomes difficult to explain so many historical episodes of hyperinflation, dollarization or money crashes. What explains then failures to implement such an easy policy? Given the tiny amount of real resources needed, fiscal baking should not be an issue. Is it just a rejection of rationality? Or in these cases authorities are simply not benevolent?

The aim of this paper is to provide a way to think about this puzzle under the lens of a trade-off that rational agents face when choosing whether or not is it worth exchanging consumption goods for money. On the one hand, money may allow agents to more efficiently save on their wealth and, on the other hand, money holdings expose agents to the risk of being taxed by the authority through seignorage. In fact, even if the authority may give value to money by offering to buy money in exchange of consumption goods at a given period, this does not imply a positive private demand of money. Agents need to evaluate the temptation of the authority to raise seignorage for its own consumption in the following period, once money is in their pockets. In such situation agents not demanding any money – *although it has real value* – is an equilibrium outcome.

To provide the argument with a formal dress, we reconsider the classical overlapping generation model of Samuelson (1958) extended as in Sims (2013) to include a storage technology with an inefficiently low real return. The presence of an alternative saving asset generates a portfolio allocation problem so that money is acquired only if its real return is not smaller than the one on storage. In this model, there exist i) a unique monetary equilibrium where money is the only store of value in the economy, ii) other suboptimal equilibria where money is used together with storage, and iii) an autarky equilibrium where storage is the only store of value. As money is used together with storage, it gradually loses value up to the autarky point where it becomes worthless. Along these equilibria consumption inequality between the young and the old increases in time as more storage is used instead of money.

In this context, we model a rational authority that has the power to tax endowments and carry out open market operations, i.e. it can use tax revenues to buy and sell money. Each period we endow the authority with a well-defined objective, which includes the utility of the agents alive in that period and its own consumption. We derive the optimal time-consistent policy and demonstrates that it selects the monetary equilibrium as the unique equilibrium. Along such equilibrium the authority does not conduct any open market operation. Nonetheless, its off-equilibrium policy, which is anticipated by the private sector, prevents any use of storage in equilibrium. In particular, in response to a (aggregate) use of storage by the young, the authority buys money to decrease the price of consumption and increase the consumption of the old. However, to maintain the same return between storage and money, the young should use more of the storage than what it is optimal from his individual point of view, making an equilibrium where storage is used unfeasible.

This result holds even if the authority gives an arbitrarily large weight to its own utility relative to agents' ones. In fact, no matter how little the authority cares about agents' utility, she always has the incentive to ensure equal consumption between the old and the young through the manipulation of market prices. Crucial to this result is the possibility to set taxes in response to aggregate savings in storage. By regulating money demand the authority maximizes total consumption, whereas by setting taxes she can keep her own consumption at the optimal level in any possible state of the world. In this sense the availability of a tax instruments contingent to aggregate states aligns private and social incentives to monetary stability: both the agents and the authority are better-off in the monetary equilibrium.

We show that the situation is radically different if we require taxes to be fixed *ex ante*.[1] Without the ability to adjust the fiscal instrument, there is a mis-alignement of incentives between the authority and the agents: manipulating the price level is

[1] As, for example, assumed in the fiscal theory of the price level.

now not only a way to equalize consumption levels across generations but also a way to gather real resources for its own consumption.

We show that, the authority is not sufficiently endowed of real resources (fixed level of taxes) or is not sufficiently benevolent, other equilibria than the monetary equilibrium exist. First, an autarky equilibrium exists in which – even with zero private money demand – the authority has an incentive to exchange consumption goods for money at a finite ratio, but she cannot resist the temptation on the following period to implicitly tax future money holders by inflating money away; as a consequence, zero private money demand self-fulfills. Second, there are other equilibria in which agents use storage and the real value of money achieves a finite stable value. In particular, facing a lack of resources, the authority has an incentive to sell money to increase consumption price and decrease consumption of the old for its own. In this case, as the young uses more of the storage the authority needs to induce more seignorage to maintain its own consumption; an equilibrium obtains when seignorage alone produces the level of inflation such that the return on money matches the return on storage without increasing the use of storage over time.

To conclude, despite at a first glance ruling out hyperinflation can be done with infinitesimal fiscal backing, the conditions for this policy to be credible in a time-consistent fashion rely heavily on which type of fiscal instrument the authority has available to back up its own desired level of consumption. The ability of the authority to tax, which is proper of a fiscal authority, sustains confidence that open market operations are indeed carried out to ensure monetary stability and not to raise seignorage, no matter the degree of benevolence of the authority's objective. On the contrary, authorities that do not have fiscal power (these can also be interpreted as private money issuers), should have a sufficiently large real endowment or a sufficiently strong interest in agents' utility to sustain an efficient use of money in the system.

Literature review. Our paper is related to the literature on the unicity of the monetary equilibrium and the determination of the price level. We have already mentioned Obstfeld and Rogoff (1983) who argue that a commitment to convert the money into a commodity is sufficient to ensure that money does not lose value in any case. A different solution is the fiscal theory of the price level, as formulated by Leeper (1991), Sims (1994), Woodford (1995) or Cochrane (2001) and Sims (2013) among others, where the government's commitment to future real surplus pins down the real value of circulating nominal debt. Both these explanations hinge on the assumptions that the government chooses first and will not deviate from its commitment or at least it is not expected to do so. In our analysis instead the authority maximizes in a time-consistent way a well-defined objective and its actions are a best reply to the actions of the private sector. Our analysis brings implications for both approaches. On the one hand, we demonstrate that the solution of Obstfeld and Rogoff (1983) does not depend on the degree of benevolence of the authority as long as it has a flexible tax instrument to secure its own consumption without relying on seignorage. In relation to the fiscal theory of the price level, our analysis clarifies that, in the presence of other storing assets, the microfoundations of the goals of the authority are crucial to ensure a positive demand for money (or government bonds); agents will switch to other saving assets in case the path of public finances does not ensure a sufficiently large return on money. In other words, portfolio optimization puts bounds on the series of fiscal surplus that can effectively pin down the price level.

More recent works about the determination of the price level includes Benigno (2017), Hagedorn (2016) or Hall and Reis (2016).[2] Benigno (2017) separates the budget constraints of the government and the one of the central bank to argue that the strategy outlined in the fiscal-theory of the price level can be implemented solely by the central bank and with a 'passive' fiscal policy, when the central bank is appropriately capitalized and can pay interests. Hall and Reis (2016) argue that the price level can be controlled by the central bank by committing to paying interest on reserves. Hagedorn (2016) shows price level determinacy in a model where the government can commit to future nominal deficits.

In contrast with these papers, we do not assume any form of commitment on the side of the fiscal/monetary authority, consistently with the discussion by Cochrane (2011) on credibility. Importantly, Cochrane (2011) clarifies that the credibility of the policymaker's actions should not be established only *on equilibrium* to show equilibrium uniqueness, but more importantly *out-of-equilibrium*, when private agents' expectations are not consistent with the central bank's desired policy. Barthlemy and Mengus (2018) investigate a similar approach but they focus on interest rate rules and they assume that currency is traded in any state of the world.

More generally, our paper is also related to works on the interaction between monetary and fiscal policy as studied by Wallace (1981), Sargent and Wallace (1981) among others. Consistently with Wallace (1981)'s irrelevance result, we show that

[2]Other references on the topic focusing on monetary rules include Loisel (2009), Atkeson et al. (2010) or Adao et al. (2011).

interventions to back money require fiscal backing. Yet, this requirement does not imply neither the fiscal theory of the price level nor, more generally, fiscal interventions at equilibrium but only out-of-equilibrium. In particular, our paper is also connected to the literature on seignorage as, for example, Bruno and Fischer (1990), that put forward the idea that monetary financing of fiscal imbalances may generate hyperinflation

Other approaches to deal with equilibrium multiplicity in monetary models have been considered. In a nutshell, the first one is legal-tender theory of money where, based on the medium of exchange approach of Kiyotaki and Wright (1989), the government can force agents to trade using a given currency (see Aiyagari and Wallace, 1997; Li and Wright, 1998, among others). The second one is the tax-theory of money as modeled by Starr (1974) among others (see Goldberg, 2012, for a recent contribution in a dynamic model).

Our paper is also related to the literature on bailouts and time-inconsistency as Schneider and Tornell (2004), Farhi and Tirole (2012) or Acharya and Yorulmazer (2007). In our setting, the *ex post* incentive to rescue money holders is *ex ante* desirable as it leads to select the monetary equilibrium. A related paper is Mengus (2017) who shows that government's bailouts may optimally be in the form of asset purchases and, when expected, such bailouts lead even intrinsically worthless to be traded at positive prices. In contrast with his approach, we investigate the impact of public interventions in selecting monetary equilibria so that these interventions are off-equilibrium.

2 A Simple Model of Fiat Money

In this section, we introduce a simple model of fiat money along the lines of Sims (2013).

Let us consider an economy populated by overlapping generations of homogenous households of unitary mass and a monetary/fiscal authority. Time is discrete and indexed by $t \in \{1, 2, ...\}$. We assume perfect foresight.

Households A each date, a new generation of homogeneous agents has born. Each agent lives two periods and then disappears. The representative agent born at time

t maximizes the following utility function:

$$U_t \equiv \log C_{y,t} + \log C_{o,t+1}, \tag{1}$$

where $C_{y,t} \geq 0$ and $C_{y,o} \geq 0$ are individual consumptions in the first and second period respectively.

Each agent born in period t is endowed with a quantity W_t of consumption good in the first period of his life. The representative young (henceforth "the young") born at time t can exchange one unit of consumption for $P_t \geq 0$ units of money or store consumption goods using a storage technology that yields a gross return $\theta < 1$ in the next period - we denote by $S_t \geq 0$ the amount of goods stored and we assume that the households cannot short-sell money, that is $M_t \geq 0$.

The budget constraint of the first period reads as:

$$C_{y,t} + \frac{M_t}{P_t} + S_t + T_{t,y} = W_t, \tag{2}$$

where M_t denotes the quantity of money purchased by the representative young at time t; $T_{t,y}$ is a tax imposed by a fiscal authority to the young at time t. Let us call $D_t \equiv S_t + M_t/P_t$ the total stock of savings.

Money has value *only* as long as can be sold in exchange of consumption, in which case money is priced. The consumption the representative old (henceforth just "the old") at time t is:

$$C_{o,t+1} = \frac{M_{t-1}}{P_t} + \theta S_{t-1} + T_{t,o}, \tag{3}$$

where P_t is the price level of consumption at time t. The first generation is born at date 0, lives just one period, has available a stock of fiat money $M_0 > 0$ does not have storage $S_0 = 0$, and has utility function $U_0 \equiv \log(C_{o,1})$.

The monetary/fiscal authority The authority controls transfers to the young ($T_{t,y}$) and to the old ($T_{t,o}$) and can also buy and store money. Let us denote by $M_{g,t}$ the amount of money held by the authority. In particular, the balance sheet of the authority satisfies:

$$T_{t,y} + \frac{M_{g,t-1}}{P_t} = \frac{M_{g,t}}{P_t} + T_{t,o} + G_t. \tag{4}$$

The left-hand side of (4) represents the resources of the authority – tax revenues and the value of her money holdings; the right-hand side collects emplacements: new money holdings and transfers to the old and government expenditures where $G_t \geq 0$. A policy $\mathcal{P}_t \equiv (T_{y,t}, M_{g,t}, G_t, T_{o,t})$ is a collection of taxes, money purchases and government expenditures that are implemented by the authority at time t.

Equilibrium Finally, let us call $\bar{M}$ the total amount of money available in the economy, which naturally satisfies at each date t:

$$\bar{M} = M_{g,t} + M_t \tag{5}$$

The definition of a market equilibrium is as follows.

Definition 1 (market equilibrium) *For a given quantity of fiat money $\bar{M}$ and a given sequence of endowements $\{W_t\}_{t=1}^{\infty}$ and policy $\{\mathcal{P}_t\}_{t=1}^{\infty}$, a market equilibrium is profiles of consumption $\{C_{y,t}\}_{t=1}^{\infty}$ and $\{C_{o,t}\}_{t=0}^{\infty}$, money holdings $\{M_t\}_{t=0}^{\infty}$, storage $\{S_t\}_{t=0}^{\infty}$, a sequence of prices $\{P_t\}_{t=1}^{\infty}$, such that, at each period $t \in \{1, 2, ...\}$:*

i) *taking prices as given, the young chooses (M_t, S_t) to maximize* (1) *s.t.* (2)-(3),

ii) *the good market clears:*

$$C_{y,t} + C_{o,t} + S_t + G_t = W_t + \theta S_{t-1}, \tag{6}$$

iii) *the financial market clears:*

$$M_{g,t} + M_t = \bar{M}. \tag{7}$$

Note that market clearing in the financial market implies that a change in the stock of money held by the authority corresponds to a opposite change in the stock held by the private sector.

Private sector optimization Let us study the problem of the representative agent. This problem is to maximize utility as stated by (1) under the constraints of (2) and (3).

We denote by ρ_t the gross per-unit real return on savings D_t. The optimal stock of savings at time t is:

$$D_t \equiv S_t + \frac{M_t}{P_t} = \frac{W_t - T_{y,t}}{2}. \tag{8}$$

for any $\rho_t = (\theta S_t + M_t/P_{t+1})/D_t$. This result is a by-product of having log utility, which greatly simplifies our algebra.

Given that M_t and S_t have to be both positive, this optimal stock of savings also implies that:

$$S_t, \frac{M_t}{P_t} \leq \frac{W_t - T_{y,t}}{2}. \tag{9}$$

The portfolio allocation between money and storage depends on the expected return on money. In particular, we have:

$$\frac{M_t}{P_t} = \frac{W_t - T_{y,t}}{2} \quad \text{and } S_t = 0 \qquad \text{if} \qquad \Pi_{t+1} < \frac{1}{\theta}, \tag{10}$$

$$\frac{M_t}{P_t} + S_t = \frac{W_t - T_{y,t}}{2} \qquad \text{if} \qquad \Pi_{t+1} = \frac{1}{\theta}, \tag{11}$$

$$S_t = \frac{W_t - T_{y,t}}{2} \quad \text{and } \frac{M_t}{P_t} = 0 \qquad \text{if} \qquad \Pi_{t+1} > \frac{1}{\theta}, \tag{12}$$

where $\Pi_{t+1} \equiv P_{t+1}/P_t$ is the inflation rate from time t to time $t+1$. The inflation rate is the inverse of the return on money. When the return on money is greater (resp. smaller) than the return on real storage, agents save everything in money (resp. storage). Money and storage may coexist only insofar yield the same return.

Using (8) we can recover the actual law of motion for inflation as:

$$\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t} = \frac{M_{t+1}}{M_t} \left(\frac{W_t - T_{y,t} - 2S_t}{W_{t+1} - T_{y,t+1} - 2S_{t+1}} \right), \tag{13}$$

that, together with (4) and (10)-(12) for any t, describe our equilibrium. Let us describe now how our market equilibrium changes depending on the policy adopted.

3 Equilibria

3.1 Absence of Policy

To isolate the role of the authority it is useful to describe the equilibrium without policy intervention, i.e. with $\mathcal{P}_t = (0, 0, 0, 0)$ at each date t. Let us start by looking at the case with a constant endowment $W_t = W$. The law for inflation becomes

$$\Pi_{t+1} = \frac{W - 2S_t}{W - 2S_{t+1}}, \tag{14}$$

given that $M_t = \bar{M}$ for any $t \geq 1$.

It is easy to note that $\Pi_{t+1} = 1 < \theta^{-1}$ and $S_t = 0$ for any t is an equilibrium; one in which money is always used and storage never. We refer to this equilibrium as the **pure monetary equilibrium**.

To check if there exist an equilibrium where storage is used jointly with money we should use the arbitrage condition (11). For $S_t > 0$ at time t we must have $\Pi_{t+1} = \theta^{-1}$. In this case, (14) obtains as

$$S_{t+1} = \theta S_t + (1 - \theta) \frac{W}{2}, \tag{15}$$

which implies $S_{t+1} \geq S_t$, given the limit $S_t \leq W/2$ for each date t. Therefore we obtain that, if storage is used in one period, it must necessarily be used on a larger extent next period. In fact, an equilibrium for each initial level of storage $S_1 \in \left[0, \frac{W}{2}\right)$ (S_0 is not an optimal choice, i.e. (15) is not an equilibrium condition for S_0) exists such that storage is always used jointly with money. It is easy to show that in the long run, storage and the real money balance satisfy:

$$\lim_{t \to \infty} S_t = \frac{W}{2} \text{ and } \lim_{t \to \infty} \frac{\bar{M}}{P_t} = 0 \tag{16}$$

for any initial level of storage S_1, where the latter obtains as a consequence of the former because of (8). There are equilibria in which storage is always used, prices grows at a rate $1/\theta$ and money looses value in time until it eventually become worthless; let us call them the **asymptotic autarky equilibria**.

Importantly, all asymptotic autarky equilibria do not necessarily feature storage at date-0 and it is possible to construct asymptotic autarky equilibria where storage is not used until a certain date s after which it is always used. In fact, notice that $S_{s-1} = 0$ only requires that $\Pi_s < \theta^{-1}$, that is

$$0 \leq S_s < (1 - \theta) \frac{W}{2}.$$

Thus, at each date t, after having only used money in past periods, it is possible to start using storage. What is peculiar of the environment with constant endowment is that once storage is used it will used for ever; this is because, for a given S_t, (15) implies a certain S_{t+1} which has the property $S_{t+1} \geq S_t$, given the limit $S_t \leq W/2$ for each t.

Finally, there also exists a **pure autarky equilibrium** defined as one in which $S_t = W/2$ and $\bar{M}/P_t = 0$ for each t in which money is never used and the price level is infinite and grows at a rate larger than $1/\theta$.

The following proposition characterizes our results.

Proposition 1 (Market Equilibrium) *For a given $\bar{M}$ and a given sequence of endowments $W_t = W$ for any $t \geq 1$ and policy $\mathcal{P}_t = (0,0,0,0)$ for any $t \geq 1$ and initial conditions $M_0 > 0$ and $S_0 = 0$ multiple equilibria exist.*

*A unique **pure monetary equilibrium** exists for*

$$P_1 = P^* \equiv \frac{2M_0}{W}$$

such that

$$\begin{aligned}
\Pi_t &= 1, \\
\frac{M_t}{P_t} &= \frac{M_0}{P^*} = \frac{W}{2}, \\
S_t &= 0
\end{aligned}$$

for any $t \geq 1$.

*An **asymptotic autarky equilibrium** exists for each $s \geq 1$ such that*

$$\begin{aligned}
\frac{M_t}{P_t} &= \frac{M_0}{P^*} = \frac{W}{2}, \text{ for any } 1 \leq t < s \\
\Pi_t &= 1, \text{ for any } 1 < t < s \\
S_t &= 0, \text{ for any } 1 < t < s
\end{aligned}$$

and $P_s > P^*$ *so that*

$$\begin{aligned} \frac{M_t}{P_t} &= \frac{M_0}{\theta^{-(t-s)}P_s} = \frac{W-\bar{T}}{2} - S_t, \text{ for any } t \geq s \\ \Pi_t &= \theta^{-1}, \text{ for any } t > s \\ S_t &= \theta^{t-(s-1)}S_s + \left(1-\theta^{t-(s-1)}\right)\frac{W}{2}, \text{ for any } t > s \\ \lim_{t\to\infty}\frac{M_t}{P_t} &= 0. \end{aligned}$$

In addition, if $s > 1$, P_s *also satisfies* $P_s \in \left(P^*, \theta^{-1}P^*\right)$.

A ***pure autarky equilibrium*** *exists where* $P_t \to \infty$, $\Pi_t > \theta^{-1}, M_t/P_t \to 0$ *and* $S_t = W/2$ *for any* $t \geq 1$.

It does not exist any equilibrium for $P_1 < P_1^*$.

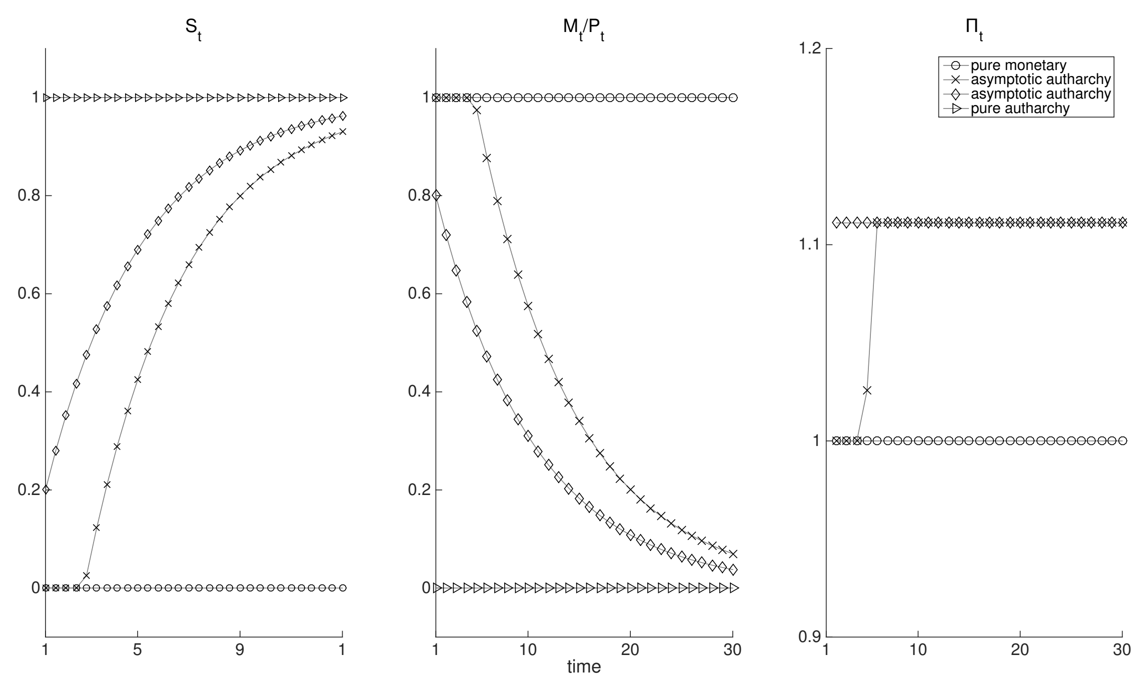

Figure 1: Equilibria without policy intervention for $\theta = 0.9, W = 2$ and $\bar{M} = 1$.

In the case of a monetary equilibrium agents perfectly equalize consumption across periods. This equilibrium, which is denoted with a circle marker in Figure 1, is characterized by a constant real value of money M_t/P_t equal to the real value of savings $W/2$, zero storage and inflation $\Pi_t = 1$, i.e. constant prices.

On the other hand any initial price level such $P_t > 2\bar{M}/W$ corresponds to an equilibrium with storage $S_t = W/2 - \bar{M}/P_t$, which implies that the real value of the private stock of money is smaller that the desired amount of real saving, which is $W/2$. As long as storage and money are used at the same time, by arbitrage we shave $\Pi_t = \theta^{-1}$ which means that, next period, the real value of money necessarily reduces and storage expands. In the end, storage converges to $\lim_{t\to\infty} S_t = W/2$. This equilibrium is denoted with a diamond marker in Figure 1.

It is worth noting that whereas there is a unique initial price associated with the unique pure monetary equilibrium P^*, there exist a continuum of equilibria with storage associated to it. In other words, even nailing down the initial price to P^* is not sufficient for having the unique pure monetary equilibrium and a single path of inflation. In Figure 1 we plot one of these. In period $t = 5$ the price level jumps to a value strictly higher than P^{ast} so that inflation rate at time $t = 5$ reaches a value between 1 and θ^{-1}, which is still compatible with an optimal private choice of having no storage at time $t = 4$. However, as the price departs from P^* necessarily there must be some positive amount of savings in storage, i.e. $S_5 > 0$; this in turns requires $\Pi_6 = \theta^{-1}$ and implies that $S_6 > S_5$. At this point the economy enters in a asymptotic autarky equilibrium.

Finally, the pure autarky equilibrium is represented with a triangle marker in Figure 1: in this case, storage is maximal, the real value of monetary savings is nul and with prices being infinitely large and growing at a rate larget than θ^{-1} (not depicted).

3.2 Optimal Policy

Policy objective

In this section, we introduce an objective for the authority and derive its optimal policy. We assume that the authority, as the agents, is active for one period only. At each date t, we assume that the government selects policies $\{\mathcal{P}_t\}_{t=1}^{\infty}$ so as to maximize the following objective function:

$$\log C_{y,t} + \log C_{o,t} + \lambda \log G_t, \tag{20}$$

that is, the authority cares about the utility of both the young and the old generations, but also the level of public expenditures, G_t weighted by a coefficient $\lambda > 0$. The case $\lambda = 0$ characterizes one in which the authority is fully benevolent; on the opposite, the case $\lambda \to \infty$ is one in which the authority is fully selfish. Note also that G_t does not necessarily entail a "waste". The $\lambda \log G_t$ component can be added to the utility of the agents without that any of our argument is affected. In such a case, G_t would denote a public good whose provision is out of the control of the agents.

We are now ready to modify the definition of an equilibrium so as to take into account the optimality of the fiscal authority's interventions:

Definition 2 *For an initial quantity of fiat money M_0 and a given sequence of endowements $W_t = W$ for any $t \geq 1$, a market equilibrium with optimal policy $\{\mathcal{P}_t\}_{t=1}^{\infty}$ is profiles of consumption $\{C_{y,t}\}_{t=1}^{\infty}$ and $\{C_{o,t}\}_{t=0}^{\infty}$, money holdings $\{M_t\}_{t=0}^{\infty}$, storage $\{S_t\}_{t=0}^{\infty}$, a sequence of prices $\{P_t\}_{t=1}^{\infty}$ and policies $\{\mathcal{P}_t^*\}_{t=1}^{\infty}$ such that:*

(i) *given policies $\{\mathcal{P}_t^*\}_{t=1}^{\infty}$, the allocation $\{C_{y,t}\}_{t=1}^{\infty}$, $\{C_{o,t}\}_{t=0}^{\infty}$, $\{M_t\}_{t=0}^{\infty}$, $\{S_t\}_{t=0}^{\infty}$ and $\{P_t\}_{t=1}^{\infty}$ is a market equilibrium.*

(ii) *at each date $t \geq 1$, date-t policy $\mathcal{P}_t^*$ maximizes (20), for given storage decisions (S_{t-1}, S_t, S_{t+1}) and given past and future policies $\left(\mathcal{P}_{t-1}^*, \mathcal{P}_{t+1}^*\right)$ for any $t > 1$ and for given (S_0, S_1, S_2) and $\mathcal{P}_2^*$ at $t = 1$.*

Note that, at each date, there is an authority responsible for the current policy and objective; in other words she cannot count on future commitment. This setup leads to a time-consistent policy. In an equivalent interpretation, the authority is infinitely living but lacks of commitment power.

Policy instruments

We first observe that, provided there is private demand of money $M_t > 0$, the authority can affect the market price by buying or supplying money for given fiscal surplus.

This can be show formally by combining the fiscal authority's budget constraint with the market clearing condition for money (7), so to obtain:

$$T_{y,t} - T_{o,t} = \frac{M_{g,t} - M_{g,t-1}}{P_t} + G_t = \frac{M_{t-1} - M_t}{P_t} + G_t. \tag{21}$$

This equality captures the open market operation: the fiscal authority can affect the market price P_t by changing the stock of money in circulation M_t by adjusting its money holdings $M_{g,t}$, holding taxes and transfers fixed. Therefore controlling $M_{g,t}$ and the fiscal surplus $T_{y,t} - T_{o,t}$ is equivalent to choosing a market price P_t.

The case in absence of private money demand $M_t = 0$ must be carefully considered. It is important to note even in this case the authority can in fact choose a price P_t, but this necessarily require acting on the fiscal surplus, that is imposing the rate of exchange between the money held by the old and net tax revenues. Note therefore, that by controlling $M_{g,t}$ and the fiscal surplus $T_{y,t} - T_{o,t}$ the authority can effectively control the price for every level of private demand for money $M_t \geq 0$.

We restrict our attention to the set of policies without transfers to old, i.e. from here onwards we look for optimal policies of the form $\mathcal{P}_t^* = (T_{y,t}^*, M_{g,t}^*, G_t^*, 0)$. Such restriction of the fiscal policy is needed to make our problem meaningful because prevents the authority from solving the saving problem of agents by directly setting the right mix of taxes and transfers. In Appendix A we show that this restriction is without loss of generality in at least in two cases. First, when there is heterogeneity in agents' discount factor but the authority lacks of individual-specific instruments, the authority would optimally set transfers to the old equal to zero. This happens because, as agents privately acquire the individually optimal stock of money, transfers operated implicitly through changes in the real value of money are more efficient than direct fiscal transfers. Second, when operating transfers occur with costs, the authority would optimally set transfers to the old equal to zero. This is because operating implicit transfer to thought the money market saves on such costs, which is intuitive. More interestingly, this remains true for whatever cost of collecting taxes, that is, there will be never optimal to resort to transfers to the old as soon as they entail a cost, no matter how small it is relative to the cost of collecting taxes.

Solving the problem of the authority

Let us state the problem of the authority as follows.

Problem 1 *At any date $t \geq 1$, an optimal policy is a $\mathcal{P}_t^* = (T_{y,t}^*, M_{g,t}^*, G_t^*, 0)$ that solves:*

$$\max_{P_t, G_t} \{\log C_{y,t} + \log C_{o,t} + \lambda \log G_t\},$$

subject to

$$T_{y,t} + \frac{M_{g,t-1}}{P_t} = \frac{M_{g,t}}{P_t} + G_t$$

taking into account agents' decision process on consumption:

$$C_{y,t} = \frac{M_t}{P_t} + S_t = \frac{W - T_{y,t}}{2} \tag{22}$$

$$C_{o,t} = \frac{M_{t-1}}{P_t} + \theta S_{t-1} \tag{23}$$

and market clearing conditions (6) *and* (7)*, with $S_0 = 0$ and $M_0 \leq \bar{M}$.*

We can make explicit the power to affect prices by plug (21) into the consumption/saving decision of the young to obtain:

$$\frac{M_t}{P_t} = W - G_t - \frac{M_{t-1}}{P_t} - 2S_t. \tag{24}$$

Such expression for current real saving can be plugged in (22), which together with (23) we use to replace consumption in the objective of the authority. The problem of the authority becomes

$$\max_{P_t, G_t} \left\{ \log \underbrace{\left(W - G_t - \frac{M_{t-1}}{P_t} - S_t \right)}_{=C_{y,t}} + \log \underbrace{\left(\frac{M_{t-1}}{P_t} + \theta S_{t-1} \right)}_{=C_{o,t}} + \lambda \log G_t \right\}, \tag{25}$$

as one depending only on current price and government's consumption. Looking into it we note that by increasing the price the authority implicitly dries resources form the young in favor of the old, and that increasing the authority's consumption further reduces consumption of the young.

Intuitively, the optimal amount of public consumption should be such that the marginal utility of consumption of the young is equal to the marginal utility of public consumption weighted by λ. Formally, the solution to this problem is given by:

$$\begin{cases} G_t = \lambda C_{y,t}, \ P_t = \frac{(2+\lambda)M_{t-1}}{W-(1+\lambda)\theta S_{t-1}-S_t} & \text{with} \quad \lim_{P_t\to\infty} C_{y,t} \geq \lim_{P_t\to\infty} C_{o,t} \\ G_t = \lambda C_{y,t}, \ P_t \to \infty & \text{otherwise.} \end{cases} \tag{26}$$

The optimal price is the price that equalizes consumption of the young with the old. A corner solution emerges when the young consumes less than the young at the autarky limit ($P_t \to \infty$). When this is the case the authority would like to choose a negative price to transfer resources from the latter to the former, which is not feasible; as a second best the authority chooses the price to be infinity; in any case its consumption remains a λ fraction of the consumption of the young.

Using (26) in (24) we get the actual laws of motion of real money and inflation:

$$\frac{M_t}{P_t} = \frac{W - (3+\lambda)\, S_t + \theta S_{t-1}}{2 + \lambda}, \tag{27}$$

$$\Pi_{t+1} = \frac{W - (3+\lambda)\, S_t + \theta S_{t-1}}{W - (1+\lambda)\theta S_t - S_{t+1}}. \tag{28}$$

provided $W > (1+\lambda)\theta S_t + S_{t+1}$, otherwise we have $M_{t+1}/P_{t+1} \to 0$ and $\Pi_{t+1} \to \infty$ is not defined. We are ready now to investigate investigate how optimally chosen policies affect equilibrium outcomes.

The pure monetary equilibrium. First, the pure monetary equilibrium where $S_t = 0$ at each t is still an equilibrium. This can be easily seen by checking that $S_t = 0$ at any t implies $\Pi_{t+1} = 1$ at any t, which are mutually consistent. Along that equilibrium we also have $M_t = M_0$, $G_t = T_{y,t} = \lambda W/(2+\lambda)$ and $C_{y,t} = C_{o,t} = W/(2+\lambda)$ at any $t \geq 1$.

Non existence of asymptotic autarky equilibria. Here we show that there are no equilibria where both money and storage are used.[3]

Indeed, suppose that one such equilibria exist. This implies that at some date t, storage is positive ($S_t > 0$) and the inflation rate satisfies $\Pi_t = \theta^{-1}$. Combined with

[3] We should note here that there could be heterogeneity in portfolio allocation between storage and money, i.e. agents may randomize. We check in Appendix B that randomization does not affect our results in no way.

the law of motion of inflation (28), this implies:

$$\theta^2 S_{t-1} - 2\theta S_t + S_{t+1} - (1-\theta)W = 0. \tag{29}$$

Let us show first that the law of motion above implies that if $S_t > 0$ then $S_{t+\tau} > 0$ for $\tau \geq 1$. First note that $S_{t-1} = 0$ and $S_t > 0$ implies

$$S_{t+1} = (1-\theta)W + 2\theta S_t > S_t > 0.$$

as $S_t < W$; hence $S_{t-1} = 0$, $S_t > 0$, $S_{t+1} = 0$ cannot be an equilibrium sequence. Then, let us check whether $S_{t-1} > 0$, $S_t > 0$, $S_{t+1} = 0$ is part of a possible solution, that is, if there exists a couple of $S_{t-1} > 0$, $S_t > 0$ such that

$$S_{t-1} = \frac{2\theta S_t + (1-\theta)W}{\theta^2}.$$

in this case, the constraint $S_{t-1} \leq W/2$ requires a negative S_t which is not possible, therefore $S_{t-1} > 0$, $S_t > 0$, $S_{t+1} = 0$ cannot be an equilibrium sequence.

Then, we can rule out the equilibrium where $S_t > 0$ for each $t \geq \tau$. To see that, note that the solution to the difference equation (29) can be rewritten in the homogeneous form:

$$\theta^2 S_{w,t-1} - 2\theta S_{w,t} + S_{w,t+1} = 0$$

with $S_{w,t} \equiv S_t - W/(1-\theta)$. This difference equation has a single real characteristic root $0 < \theta < 1$, so that the sequence $S_{w,t}$ converges monotonically to 0, for any initial conditions. As a consequence, S_t converges to $\bar{S} = W/(1-\theta)$. However, this contradicts that the maximal storage is $S_t = W/2 \leq \bar{S}$ and therefore asymptotic autarky equilibria do not exist.

Non existence of a pure autarky equilibrium. Here we prove that an equilibrium in which real money balance are always valueless ($M_t/P_t = 0$ for any t) do not exist. In such an equilibrium, date-1 real money balance held by private agents also satisfies $M_1/P_1 = 0$. Then the government budget constraint and the financial market clearing condition imply that $T_{y,1} = M_0/P_1 + G_1$. By substituting the latter into the optimal autarky decision $S_1 = (W - T_{y,1})/2$, we get that the storage compatible with $M_1/P_1 = 0$ is

$$S_1 = \bar{S}(S_0) \equiv \frac{W + \theta S_0}{3 + \lambda}, \tag{30}$$

where, note, $\bar{S}(x) \leq W/2$ as $x \leq W/2$. Plugging this into (26), which still holds, for initial conditions $M_0 > 0$ and $S_0 = 0$we get that the price in the first period is

$$\tilde{P}_1 = \frac{3 + \lambda}{W} M_0$$

which is finite and positive. Hence $P_1 = \infty$ is not a solution, the authority would always like to exchange money held by the old for consumption good that she collect by raising taxes. Therefore, $M_1/P_1 = 0$ can happen only with $M_1 = 0$, however $M_1 = 0$ cannot be an equilibrium. To see this, note that, in this case, the price at time 2 would be

$$P_2 = \frac{(2 + \lambda) M_1}{W - (1 + \lambda)\theta \bar{S}(0) - S_2} \geq 0$$

which is finite and positive, even for the maximal storage at period 2, namely $S_2 = \bar{S}(\bar{S}(0))$ (in which case $M_2 = 0$). The important observation is that with $M_1 = 0$ and $S_1 = \bar{S}(0)$ necessarily $\Pi_2 = P_2/\tilde{P}_1 = 0$ irrespective of M_2. However, $\Pi_2 = 0$ is not compatible with private storage choice $S_1 \neq 0$. The return on money would then be $+\infty$, which obviously exceeds the one on storage θ. The same rasoning applies at any t for $S_{t-1} = 0$. Therefore we conclude that an equilibrium where $M_1/P_1 = 0$ is not possible, and as a consequence, a pure autarky equilibrium does not exist.

The following proposition summarizes all these findings:

Proposition 2 *For a given $\lambda \geq 0$, a given $\bar{M}$, a given sequence of endowements $W_t = W$ for any $t \geq 1$ and policy $\mathcal{P}_t = \mathcal{P}_t^*$ for any $t \geq 1$ and initial conditions $M_0 > 0$ and $S_0 = 0$, a unique equilibrium exists. In such equilibrium, $P_1 = P^*$ and*

(i) there is no inflation $\Pi_t = 1$,

(ii) the real value of money is equal to the desired level of real savings

$$\frac{M_t}{P_t} = \frac{W}{2 + \lambda} \quad \text{and} \quad S_t = 0,$$

(iii) there are no public open market interventions:

$$T_{y,t} = G_t = \frac{\lambda}{2+\lambda} W,$$

for each $t \geq 1$.

The only equilibrium outcome when policy is optimally chosen is a pure monetary equilibrium. Importantly, in this equilibrium, the policy does not step in and raises no taxes. This means that the value of money does not derive from *in-equilibrium* taxation but from its liquidity services. The role of the taxation power is only *out-of-equilibrium* to rule out undesired equilibrium paths.

Note also that the proposition is independent of the weight on government's expenditures, λ. The key intuition is that the consumption of the government is a fraction of the consumption of the old (which is equal to one of the young), who is better off in an economy where money has value. As a result, whatever the value of λ, the government always prefers the economy to stay in the monetary equilibrium where everyone is better off.

Monetary policy with fixed fiscal baking

Let us now investigate the situation where the fiscal authority cannot raise taxes. We will restrict further our class of policies to $\hat{\mathcal{P}}_t = (\bar{T}, M^*_{g,t}, G^*_t, 0)$ where taxes on the young $T_{y,t} = \bar{T}$ are taken fixed through time. As a result, it has to rely only on seignorage to fund its expenditures. To intervene, the authority can then adjust its expenditures to reduce seignorage. Our policy specification of no taxes on the old is not restriction as in this section we do not consider taxes as controls.

More specifically, the fiscal authority solves the following problem:

Problem 2 *At date t, the authority solves:*

$$\max_{M_{g,t}, G_t} \left\{ \log C_{y,t} + \log C_{o,t} + \lambda \log G_t \right\},$$

subject to

$$\bar{T} + \frac{M_{g,t-1}}{P_t} = \frac{M_{g,t}}{P_t} + G_t$$

taking into account agents' decision process on consumption:

$$C_{y,t} = \frac{M_t}{P_t} + S_t = \frac{W - \bar{T}}{2} \text{ and } C_{o,t} = \frac{M_{t-1}}{P_t} + \theta S_{t-1}$$

and market clearing conditions (6) *and* (7).

As before, by combining the fiscal authority's budget constraint with the market clearing condition for money, (7), we obtain:

$$\bar{T} = \frac{M_{g,t} - M_{g,t-1}}{P_t} + G_t = \frac{M_{t-1} - M_t}{P_t} + G_t. \tag{31}$$

In this case, controlling $M_{g,t}$ and G_t given $T_{y,t} = \bar{T}$ fixed is equivalent to choosing a market price P_t. It is important to note that the authority can still choose a price level by simply imposing the rate of exchange between the money held by the old and by adjusting its expenditures, despite the fact that the taxes collected by the authority are fixed.

We can use the budget constraint of the authority to eliminate M_t/P_t from the optimal private saving choice and obtain an expression for G_t. We can then rewrite the problem of the authority as

$$\max_{P_t} \left\{ \log \frac{W - \bar{T}}{2} + \log \underbrace{\left(\frac{M_{t-1}}{P_t} + \theta S_{t-1} \right)}_{=C_{o,t}} + \lambda \log \underbrace{\left(\frac{W + \bar{T}}{2} - \frac{M_{t-1}}{P_t} - S_t \right)}_{=G_t} \right\}. \tag{32}$$

The solution to this problem is:

$$\begin{cases} P_t = \frac{2(1+\lambda)M_{t-1}}{W + \bar{T} - 2\lambda\theta S_{t-1} - 2S_t} & \text{with} \quad \lim_{P_t \to \infty} C_{o,t} \leq \lim_{P_t \to \infty} G_t \\ P_t \to \infty & \text{otherwise.} \end{cases} \tag{33}$$

The market price for money is an instrument now to indirectly tax the old to get consumption for the authority. By loosing the ability to tax the authority looses the ability to influence the demand of savings and so the consumption of the young. Therefore whereas before the price was an instrument to equate consumption of the agents, it is now an instrument to implicitly tax. A corner solution emerges here at the point where the authority would be still beneficial to transfer resources from the old to the authority, but the price already exhausted its scope.

Using (31) and (33), we get the actual law of motion of inflation of the real value of savings and inflation as:

$$\frac{M_t}{P_t} = \frac{W-\bar{T}}{2} - S_t \tag{34}$$

$$\Pi_{t+1} = \frac{P_{t+1}}{P_t} = \frac{(1+\lambda)\left(W+\bar{T}\right) - 2(1+\lambda)S_t}{W+\bar{T} - \ 2\lambda\theta S_t - 2S_{t+1}} \tag{35}$$

provided $W+\bar{T} \geq 2\lambda\theta S_t + 2S_{t+1}$, otherwise we have $M_{t+1}/P_{t+1} \to 0$ and $\Pi_{t+1} \to \infty$. We are ready now to investigate investigate how optimally chosen policies affect equilibrium outcomes.

The pure monetary equilibrium. The pure monetary equilibrium where $S_t = 0$ at each t is still an equilibrium provided $1+\lambda < \theta^{-1}$. This can be easily seen by checking that $S_t = 0$ at any t implies $\Pi_{t+1} = 1+\lambda$ at any t from (35). In turn, $S_t = 0$ requires that $\Pi_{t+1} \leq \theta^{-1}$, thus implying that $1+\lambda$ does not exceed θ^{-1}. Along that equilibrium, money is growing at a rate $1+\lambda$:

$$M_t = (1+\lambda)^t M_0. \tag{36}$$

Government expenditures are financed through taxes and seignorage:

$$G_t = \frac{\lambda}{1+\lambda}\frac{W+\bar{T}}{2}. \tag{37}$$

Finally, private consumption satisfies:

$$C_{y,t} = \frac{W-\bar{T}}{2} \text{ and } C_{o,t} = \frac{W+\bar{T}}{2(1+\lambda)} \text{ at any } t \geq 1. \tag{38}$$

In case $\Pi_{t+1} > \theta^{-1}$ implies $S_t > 0$, so that a pure monetary equilibrium does not exist in that case.

Existence of asymptotic storage equilibria. We investigate now whether there are equilibria where both money and storage are used. $S_t > 0$ implies $\Pi_t = \theta^{-1}$ at t

that, is

$$S_{t+1} = \theta S_t + (1-(1+\lambda)\theta)\frac{W+\bar{T}}{2}.$$

Let us first consider the case $1 > (1+\lambda)\theta$. In such a case, $S_t > 0$ implies $S_{t+\tau} > 0$ for $\tau \geq 1$. However, an equilibrium where $S_t > 0$ for each $t \geq \tau$ requires a sequence $\{S_t\}_{t=1}^{\infty}$ converging monotonically to

$$\bar{S} = \frac{1-(1+\lambda)\theta}{1-\theta}\frac{W+\bar{T}}{2}.$$

As previously noted, to be feasible, $\bar{S}$ should satisfy $\bar{S} \leq \left(W-\bar{T}\right)/2$. As a result, a necessary condition to be an equilibrium is:

$$\bar{T} < \frac{\theta\lambda}{2-(2+\lambda)\,\theta}W.$$

Otherwise, an equilibrium where money and storage are jointly used does not exist.

Similarly to the case without any policy, all asymptotic storage equilibria do not necessarily feature storage at date-0 and it is possible to construct asymptotic storage equilibria where storage is not used until a certain date s after which it is always used. In fact, notice that $S_{s-1} = 0$ only requires that $\Pi_s \leq \theta^{-1}$, that is

$$0 \leq S_s < (1-(1+\lambda)\theta)\frac{W+\bar{T}}{2}$$

Thus, at each date t, after having only used money in past periods, it is possible to start using storage. Also here once storage is used it will used for ever.

Finally, in the case when $1 < (1+\lambda)\theta$, the sequence of storage S_t converges to a negative value; however this violates the constraint $S_t \geq 0$. Thus, in this case, an equilibrium where storage is used with money does not exist.

Existence of pure autarky equilibria. We study here the conditions for the existence of a pure autarky equilibrium – i.e. one in which $M_t/P_t = 0$ for any t. As before we look at the initial period. Suppose that $M_1/P_1 = 0$. Then $\bar{T} = M_0/P_1 + G_1$ so that (33) still hold at $t = 1$. The storage compatible with $M_t/P_t = 0$ is

$$S_1 = \frac{W-\bar{T}}{2}.$$

Plugging this into (33), for initial conditions $M_0 > 0$ we get that, in this case, the price level in the first period has to satisfy:

$$\underline{P}_1 = \frac{1+\lambda}{\bar{T}} M_0$$

which is positive and finite provided $\bar{T} > 0$. Hence $P_1 = \infty$ is not a solution as long as $\bar{T} > 0$, in this case the authority would always like to exchange money held by the old for consumption good that she collects by raising taxes. Thus, with $\bar{T} > 0$, autarky $M_1/P_1 = 0$ can happen only with $M_1 = 0$, however can be $M_1 = 0$ a solution?

Suppose that $M_1 > 0$. In this case, the price at time 2 is:

$$P_2 = \frac{2(1+\lambda)}{(1-\lambda\theta)W + (1+\lambda\theta)\bar{T} - 2S_2} M_1$$

which is not always positive and finite for maximal storage $S_2 = \left(W - \bar{T}\right)/2$. In particular, $P_t = \infty$ for each t is a possible equilibrium outcome when:

$$\bar{T} < \frac{\lambda\theta}{2+\lambda\theta} W,$$

Only in such a case a pure autarky equilibria exists.

The following proposition summarizes all these findings:

Proposition 3 *For a given $\bar{M}$ and a given sequence of endowements $W_t = W$ for any $t \geq 1$ and policy $\mathcal{P}_t = \hat{\mathcal{P}}_t$ for any $t \geq 1$ and initial conditions $M_0 > 0$ and $S_0 = 0$, multiple market equilibrium exist for any $\lambda \geq 0$.*

(i) *Provided that $1 + \lambda \leq \theta^{-1}$, a unique* ***pure monetary equilibrium*** *such that:*

$$\begin{aligned} \frac{M_t}{P_t} &= \frac{M_0}{P^*} = \frac{W - \bar{T}}{2}, \text{ for any } t \geq 1, \\ \Pi_t &= 1 + \lambda, \text{ for any } t > 1, \\ S_t &= 0, \text{ for any } t > 1. \end{aligned}$$

(ii) *Provided that* $1+\lambda \leq \theta^{-1}$ *and*

$$\bar{T} < \frac{\theta\lambda}{2-(2+\lambda)\,\theta} W,$$

an ***asymptotic storage equilibrium*** *exists for each* $s \geq 1$ *such that*

$$\begin{aligned} \frac{M_t}{P_t} &= \frac{M_0}{P^*} = \frac{W-\bar{T}}{2}, \text{ for any } 1 \leq t < s \\ \Pi_t &= 1+\lambda, \text{ for any } 1 < t < s \\ S_t &= 0, \text{ for any } 1 < t < s \end{aligned}$$

and $P_1 \geq P^*$ *and* $P_s \geq (1+\lambda)^{s-1} P^*$ *so that:*

$$\begin{aligned} \frac{M_t}{P_t} &= \frac{W-\bar{T}}{2} - S_t, \text{ for any } t \geq s \\ \Pi_t &= \theta^{-1}, \text{ for any } t > s \\ S_t &= \theta^{t-(s-1)} S_s + \frac{1-\theta^{t-(s-1)}}{1-\theta}(1-(1+\lambda)\theta)\frac{W+\bar{T}}{2}, \text{ for any } t > s \\ \lim_{t\to\infty} \frac{M_t}{P_t} &= \frac{\theta\lambda}{1-\theta}\frac{W+\bar{T}}{2} - \bar{T} \geq 0. \end{aligned}$$

In addition, if $s > 1$, $P_s \in \left((1+\lambda)^{s-1}P^*, (1+\lambda)^{s-1}\theta^{-1}P^*\right)$.

(iii) *When*

$$\bar{T} < \frac{\lambda\theta}{2+\lambda\theta} W,$$

a ***pure autarky equilibrium*** *exists where* $P_t \to \infty$, $\Pi_t > \theta^{-1}, M_t = 0$ *and* $S_t = W/2$ *for any* $t \geq 1$.

It does not exist any equilibrium for $P_1 < P^*$.

The proposition is illustrated by Figure 2. In contrast to the case plotted in Figure 1, equilibria in which storage is used jointly with money converge to a situation in which the real value of monetary savings and storage reache a steady state level. This equilibria exist with sufficiently small fiscal baking, in which case also pure autarky equilibria exist.

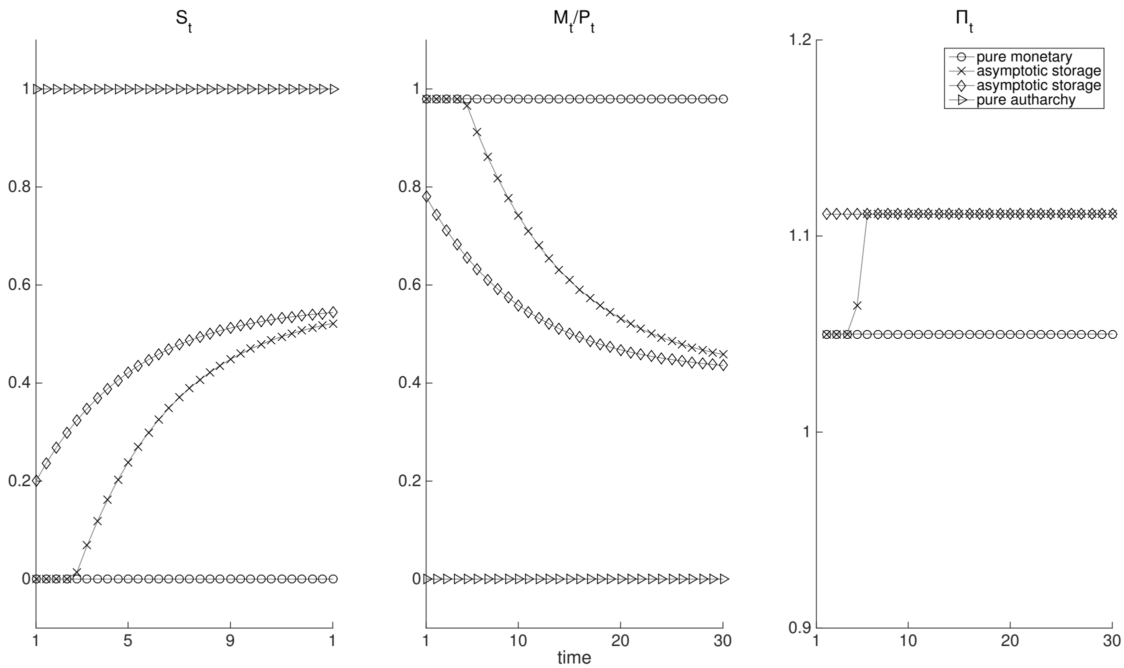

Figure 2: Equilibria with fixed backing for $\theta = 0.9, W - \bar{T} = 2$, $\bar{M} = 1$ and $\lambda = 0.05$.

The set of equilibria crucially depends on the level of fix fiscal baking, as the impossibility to use taxes as a contingent instrument creates incentives to use money purchases as an instrument to tax the old.

This trade-off does not arise when taxes can be freely set, as then the government has sufficient tools to adjust government expenditures. In such a case, the authority ensures the value of money to improve total available consumption goods and taxes to ensure the fraction that it needs. In contrast, when taxes are fixed, the government can only adjust expenditures to purchase money and, thus, it trades off the welfare gains of money trading with its cost of cutting expenditures.

When taxes are too low and, thus, government expenditures are low as well, the government may even have the incentive to drive the price level to negative values so as to tax money holdings. Yet, negative price levels are not feasible but such an incentive prevents money trading and allows autarky to be an equilibrium outcome.

Overall, the main reason that committing to taxes does not ensure the unicity of the equilibrium as, for example, in Sims (2013) is that agents are not forced to hold money: households can hold no money ($M_t = 0$). In contrast, when agents necessarily hold all the money, as it is generally assumed in the fiscal theory of the price level, it is sufficient to pin down the real value of the stock of money in circulation to select an equilibrium but this assumes away that the stock of money in circulation is also a private agents' decision.

A Micro-foundations for money purchases

In this appendix, we investigate some motives that make money purchases preferred to direct transfers to old households.

Preference heterogeneity To begin with, agents can differ in their preferences. This can translate into heterogenous savings. Let us elaborate an example of such heterogeneity.

Let us assume that agents' preferences dare as follows: $u(c_O, c_Y) = \log c_Y + \gamma_i \log c_0$ with heterogenous γ_i. We also assume that a group of mass p of agents are such that $\gamma_i = 1$ – *savers* – and the rest are such that $\gamma_i = 0$ – *consumers*. The former agents save half of their endowment net of taxes to be consumed in the second period of their life – as in the benchmark model – , while the latter do not save at all.

As a result, their consumption while being young are:

$$c_{y,t}^S = \frac{M_t^S}{P_t} + S_t^S = \frac{W - T_{y,t}}{2} \text{ and } c_{y,t}^C = W - T_{y,t}, \tag{40}$$

where $c_{y,t}^S$ is the consumption of savers and $c_{y,t}^S$ the consumption of consumers.

The government's budget constraint is:

$$T_{t,y} = \frac{M_{t-1} - M_t}{P_t} - T_{o,t} \tag{41}$$

and, thus:

$$c_{y,t}^S = \frac{M_t^S}{P_t} + S_t^S = \frac{W - \frac{M_{t-1} - M_t}{P_t} + T_{o,t}}{2} \text{ and } c_{y,t}^C = W - \frac{M_{t-1} - M_t}{P_t} + T_{o,t}. \tag{42}$$

Integrating the first equality across all savers yields:

$$\frac{M_t}{P_t} = \frac{p}{2-p}\left(W - \frac{M_{t-1}}{P_t} + T_{o,t}\right) - \frac{2}{2-p} S_t \tag{43}$$

We can plug this value into the expressions for agents' consumptions levels so that the current stock of money M_t disappears:

$$c_{y,t}^S = \frac{M_t^S}{P_t} + S_t^S = \frac{1}{2-p}\left(W - \frac{M_{t-1}}{P_t} + T_{o,t} - S_t\right) \tag{44}$$

$$c_{y,t}^C = \frac{2}{2-p}\left(W - \frac{M_{t-1}}{P_t} + T_{o,t} - S_t\right) = 2c_{y,t}^S \tag{45}$$

The resulting problem for the authority is:

$$\max_{P_t, T_{o,t}} \left\{ \int \log\left(c^i_{y,t}\right) \mathrm{di} + \int \log\left(\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1} - T_{o,t}\right) \mathrm{di} \right\}. \tag{46}$$

The first order conditions with respect to P_t and $T_{o,t}$ are as follows:

$$M_{t-1} \frac{1}{W - \frac{M_{t-1}}{P_t} + T_{o,t} - S_t} = \int \frac{\gamma_i M_{i,t-1}}{\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1} - T_{o,t}} \tag{47}$$

$$\frac{1}{W - \frac{M_{t-1}}{P_t} + T_{o,t} - S_t} = \int \frac{\gamma_i}{\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1} - T_{o,t}} \tag{48}$$

with $c^S_{y,t}$ and $c^C_{y,t}$ defined by equations (44) and (45).

Let us compute the right hand sides of the two conditions:

$$\int \frac{\gamma_i M_{i,t-1}}{\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1} - T_{o,t}} = \frac{M_{t-1}}{1/p\left(\frac{M_{t-1}}{P_t} + \theta S_{t-1}\right) - T_{o,t}} \tag{49}$$

$$\int \frac{\gamma_i}{\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1} - T_{o,t}} = \frac{p}{1/p\left(\frac{M_{t-1}}{P_t} + \theta S_{t-1}\right) - T_{o,t}} \tag{50}$$

The first order conditions can then be written:

$$\frac{1}{W - \frac{M_{t-1}}{P_t} + T_{o,t} - S_t} = \frac{1}{1/p\left(\frac{M_{t-1}}{P_t} + \theta S_{t-1}\right) - T_{o,t}} \tag{51}$$

$$\frac{1}{W - \frac{M_{t-1}}{P_t} + T_{o,t} - S_t} = \frac{p}{1/p\left(\frac{M_{t-1}}{P_t} + \theta S_{t-1}\right) - T_{o,t}} \tag{52}$$

Both conditions cannot hold at the same time as soon as $p < 1$, which implies that only:

$$\frac{1}{W - \frac{M_{t-1}}{P_t} - S_t} = \frac{p}{\left(\frac{M_{t-1}}{P_t} + \theta S_{t-1}\right)} \tag{53}$$

may bind in equilibrium. In particular, that means that there exists no interior solution for $T_{o,t}$ that has to equal 0. As a result of these conditions, we obtain the following expression for M_{t-1}/P_t:

$$\frac{M_{t-1}}{P_t} = \frac{pW - \theta S_{t-1} - pS_t}{1 + p}, \tag{54}$$

which allows to also rewrite M_t/P_t as follows:

$$\frac{M_t}{P_t} = \frac{1}{(2-p)(1+p)} \left(pW + \theta p S_{t-1} + (p^2 - 2(1+p))S_t\right). \tag{55}$$

The inflation rate at $t+1$ can be expressed as function of storage. Using the no-arbitrage condition between money and storage, we find:

$$\frac{pW + \theta p S_{t-1} + (p^2 - 2(1+p))S_t}{pW - \theta S_t - pS_{t+1}} \frac{1}{2-p} = \theta^{-1} \tag{56}$$

which leads to:

$$(2-p-\theta)W = (2-p)S_{t+1} + \theta(p-3)S_t + \theta^2 S_{t-1}. \tag{57}$$

As in the benchmark case, the sequences S_t satisfying this equation are of the following form, for $p < 1$:

$$S_t = \lambda_1 \theta^t + \lambda_2 \left(\frac{\theta}{2-p}\right)^t + \frac{2-p-\theta}{2-p-\theta+\theta(p-2)+\theta^2} W \tag{58}$$

As θ and $\theta/(2-p)$ are both below 1, S_t converges to $\frac{2-p-\theta}{2-p-\theta+\theta(p-2)+\theta^2}W$. Given that $\theta(p-2)+\theta^2 = \theta(\theta+p-2) < 0$, we then obtain that S_t is ultimately above $W/2$. We can then use the same logic as for the proof of Proposition 2.

General conditions for having money purchases Let us investigate more the conditions under which money purchases are preferred to direct transfers. To do so, let us introduce two costs in our benchmark model. First, the cost of transferring $T_{o,t}$ to old agents is $(1+\nu)T_{o,t}$. Second, we assume that the cost of raising $T_{y,t}$ amount of resources cost $(1+\lambda)T_{y,t}$ to young agents.

We plug the government budget constraint $T_{t,y} = \frac{M_{t-1}-M_t}{P_t} + (1+\nu)T_{o,t}$ into individual saving decisions:

$$c^i_{y,t} = \frac{M_{i,t}}{P_t} + S_{i,t} = \frac{W - (1+\lambda)T_{y,t}}{2} \tag{59}$$

to obtain these individual saving decisions as follows:

$$2\frac{M_{i,t}}{P_t} + 2S_{i,t} = W - (1+\lambda)\frac{M_{t-1}}{P_t} + (1+\lambda)\frac{M_t}{P_t} + (1+\lambda)(1+\nu)T_{o,t} \tag{60}$$

Integrated over i, this condition yields:

$$\frac{M_t}{P_t} = \frac{W - (1+\lambda)\frac{M_{t-1}}{P_t} + (1+\lambda)(1+\nu)T_{o,t} - 2S_t}{1-\lambda} \tag{61}$$

and, thus:

$$c_{y,t}^i = \frac{M_{i,t}}{P_t} + S_{i,t} = \frac{W - (1+\lambda)\frac{M_{t-1}}{P_t} + (1+\lambda)(1+\nu)T_{o,t} - (1+\lambda)S_t}{1-\lambda} \tag{62}$$

Except for this, agents are homogenous. The problem can be rewritten as:

$$\max_{P_t,T_o} \int \log\left(\frac{W - (1+\lambda)\frac{M_{t-1}}{P_t} + (1+\lambda)(1+\nu)T_{o,t} - (1+\lambda)S_t}{1-\lambda}\right) \mathrm{di} + \cdots \tag{63}$$

$$\cdots + \int \log\left(\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1} - T_{o,t}\right) \mathrm{di}, \tag{64}$$

The first order conditions with respect to P_t and $T_{o,t}$ are:

$$\frac{M_{t-1}(1+\lambda)}{W - (1+\lambda)\frac{M_{t-1}}{P_t} + (1+\lambda)T_{o,t}(1+\nu) - (1+\lambda)S_t} = \frac{M_{t-1}}{\frac{M_{t-1}}{P_t} + \theta S_{t-1} - T_{o,t}} \tag{65}$$

$$\frac{(1+\nu)(1+\lambda)}{W - (1+\lambda)\frac{M_{t-1}}{P_t} + (1+\lambda)T_{o,t}(1+\nu) - (1+\lambda)S_t} = \frac{1}{\frac{M_{t-1}}{P_t} + \theta S_{t-1} - T_{o,t}} \tag{66}$$

These two conditions cannot hold at the same time as soon as $\nu > 0$. In particular, the first constraint always bind while the second binds only when $\nu = 0$, thus implying that $T_{o,t} = 0$. Interestingly, the cost of raising taxes on the young, λ has a symmetric effect on the two conditions, indicating that direct transfers can be ruled out not because of the cost of raising resources but because of the relative cost of transfers over money purchases, as captured by ν. In the end, when the cost of raising resources satisfies $\lambda = 0$ as in the benchmark model, the optimality condition for money purchases leads to the same first order condition as (**??**).

In the end, money purchases are preferred to direct transfers only when the cost of transfers to the old (ν) is positive but not when only the cost of transfers to the young (λ) is positive. This then implies that the frictions that lead to money purchases have to increase to cost of transferring resources but not the cost of raising resources, which affects both direct transfers and money purchases.

B Randomization of portfolios

In the case where agents are indifferent between storage and money, they may randomize portfolios so that these portfolios are heterogeneous. In this appendix, we show that such a randomization does not affect our results.

First, let us find the consumption level of a young agent i. To this end, we plug the government budget constraint $T_{t,y} = \frac{M_{t-1}-M_t}{P_t}$ into individual saving decisions:

$$c_{y,t}^{i} = \frac{M_{i,t}}{P_t} + S_{i,t} = \frac{W - T_{y,t}}{2} \tag{67}$$

to obtain these individual saving decisions as follows:

$$2\frac{M_{i,t}}{P_t} + 2S_{i,t} = W - \frac{M_{t-1}}{P_t} + \frac{M_t}{P_t} \tag{68}$$

Integrated over i, this condition yields:

$$\frac{M_t}{P_t} = W - \frac{M_{t-1}}{P_t} - 2S_t \tag{69}$$

and, thus:

$$c_{y,t}^{i} = \frac{M_{i,t}}{P_t} + S_{i,t} = W - \frac{M_{t-1}}{P_t} - S_t \tag{70}$$

This leads to the following optimization problem:

$$\max_{P_t} \left\{ \int \log\left(W - \frac{M_{t-1}}{P_t} - S_t\right) \mathrm{di} + \int \log\left(\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1}\right) \mathrm{di} \right\}, \tag{71}$$

Note that the young generation consume the same, no matter its portfolio choice, consistently with young agents' indifference between portfolios.

The first order conditions with respect to P_t is:

$$\frac{M_{t-1}}{W - \frac{M_{t-1}}{P_t} - S_t} = \int \frac{M_{i,t-1}}{\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1}} \tag{72}$$

Interestingly, (72) can be rewritten in a more compact way:

$$cov\left(M_{i,t-1}, \frac{1}{\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1} - T_{o,t}}\right) = 0 \tag{73}$$

In equilibrium, if agents are indifferent between storage and money, a no-arbitrage condition should hold on asset returns: $\theta = P_{t-1}/P_t$. Using (67), this implies that

$$\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1} = \theta \left(\frac{M_{i,t-1}}{P_{t-1}} + S_{i,t-1} \right) = \theta \frac{W - T_{y,t}}{2}, \tag{74}$$

which implies that $\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1}$ is constant across individuals. Integrating this condition over households and using the fact that there is a mass 1 of them, we obtain that:

$$\frac{M_{i,t-1}}{P_t} + \theta S_{i,t-1} = \frac{M_{t-1}}{P_t} + \theta S_{t-1}. \tag{75}$$

As a result, in equilibrium, equation (72) simplifies so that we obtain the same first order condition as in the homogenous case:

$$\frac{M_{t-1}}{W - \frac{M_{t-1}}{P_t} - S_t} = \frac{M_{t-1}}{\frac{M_{t-1}}{P_t} + \theta S_{t-1}} \tag{76}$$

References

Acharya, V. V. and T. Yorulmazer (2007): "Too many to fail–An analysis of time-inconsistency in bank closure policies," *Journal of Financial Intermediation*, 16, 1–31.

Adao, B., I. Correia, and P. Teles (2011): "Unique Monetary Equilibria with Interest Rate Rules," *Review of Economic Dynamics*, 14, 432–442.

Aiyagari, S. R. and N. Wallace (1997): "Government Transaction Policy, the Medium of Exchange, and Welfare," *Journal of Economic Theory*, 74, 1–18.

Atkeson, A., V. V. Chari, and P. J. Kehoe (2010): "Sophisticated Monetary Policies," *The Quarterly Journal of Economics*, 125, 47–89.

Barthlemy, J. and E. Mengus (2018): "Nominal Anchoring, Disanchoring and Reanchoring: The role of Credibility," Mimeo Banque de France and HEC Paris.

Benigno, P. (2017): "A Central Bank Theory of Price Level Determination," CEPR Discussion Papers 11966, C.E.P.R. Discussion Papers.

Bruno, M. and S. Fischer (1990): "Seigniorage, Operating Rules, and the High Inflation Trap," *The Quarterly Journal of Economics*, 105, 353–374.

Cochrane, J. H. (2001): "Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level," *Econometrica*, 69, 69–116.

——— (2011): "Determinacy and Identification with Taylor Rules," *Journal of Political Economy*, 119, 565–615.

Farhi, E. and J. Tirole (2012): "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *American Economic Review*, 102, 60–93.

Goldberg, D. (2012): "The tax-foundation theory of fiat money," *Economic Theory*, 50, 489–497.

Hagedorn, M. (2016): "A Demand Theory of the Price Level," CEPR Discussion Papers 11364, C.E.P.R. Discussion Papers.

Hall, R. E. and R. Reis (2016): "Achieving Price Stability by Manipulating the Central Bank's Payment on Reserves," CEPR Discussion Papers 11578, C.E.P.R. Discussion Papers.

Kiyotaki, N. and R. Wright (1989): "On Money as a Medium of Exchange," *Journal of Political Economy*, 97, 927–954.

Leeper, E. M. (1991): "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, 27, 129–147.

Li, Y. and R. Wright (1998): "Government Transaction Policy, Media of Exchange, and Prices," *Journal of Economic Theory*, 81, 290–313.

Loisel, O. (2009): "Bubble-free policy feedback rules," *Journal of Economic Theory*, 144, 1521–1559.

Mengus, E. (2017): "Asset Purchase Bailouts and Endogenous Implicit Guarantees," Tech. rep., HEC Paris, research Paper No. ECO/SCD-2018-1248.

Obstfeld, M. and K. Rogoff (1983): "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy*, 91, 675–687.

Samuelson, P. A. (1958): "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 66, 467–482.

Sargent, T. J. and N. Wallace (1981): "Some unpleasant monetarist arithmetic," *Quarterly Review*.

Schneider, M. and A. Tornell (2004): "Balance Sheet Effects, Bailout Guarantees and Financial Crises," *Review of Economic Studies*, 71, 883–913.

Sims, C. A. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4, 381–399.

——— (2013): "Paper Money," *American Economic Review*, 103, 563–584.

Starr, R. M. (1974): "The Price of Money in a Pure Exchange Monetary Economy with Taxation," *Econometrica*, 42, 45–54.

Wallace, N. (1981): "A Modigliani-Miller theorem for open-market operations," *The American Economic Review*, 71, 267–274.

Woodford, M. (1995): "Price-level determinacy without control of a monetary aggregate," *Carnegie-Rochester Conference Series on Public Policy*, 43, 1–46.